



An Exploration on Graph Coloring Behaviour and Methods

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Abstract— Graph coloring is the critical and the complex algorithmic problem used with different configuration and constraints for different applications. This paper has explored the graph coloring method in different aspects. Some of the new measures, methods and constraints are also defined for adaptive graph coloring. The theorem specific graph coloring is here defined to provide the constraint generation and mapping based on which the colorization can be improved. Different features and the rule formulation for dynamic graph coloring with color specification are provided in this paper.

Keywords: Graph Coloring, Constraint, Behavior, Methods, Measures

I. INTRODUCTION

Graph Coloring is the benchmark problem defined for effective resource allocation and utilization. According to the applications and behaviors, different forms of graph coloring exist with different constraint specification. In the common and the benchmark theory of graph coloring, there is requirement to assign the C colors to all N nodes of graph G . Here $G (V, E)$ is the graph with two sets V and E . Set V is the Set of vertices and represented by $V (G)$ and E is the set of edges represented by $E (G)$. Each edge is the connectivity between two vertices v_1 and v_2 . For graph coloring, it is required that the $\text{Color}(v_1) \neq \text{Color}(v_2)$, if there exist edge between v_1 and v_2 . This kind of graph coloring with standard phenomenon is called proper coloring. The challenge in graph coloring is to utilize the minimum number of colors and identify the solution in effective time. When the number of possible colors in the graph coloring is already defined called k -colors then the coloring is known as k -colorable. Here figure 1 is showing the proper coloring.

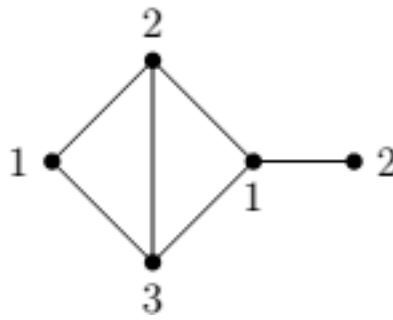


Figure 1: Proper Coloring

Here figure 1 is showing the proper coloring method with 3 colors. The number of colors used in graph coloring is called chromatic number. Graph coloring can be represented in different forms. Some of these coloring representation methods are described in this section

A) List Coloring

List coloring is the generalized coloring method in which each of nodes represents the list of available colors. Based on the further analysis, the acceptability of color is decided and final color is assigned. The list coloring behavior and representation is shown here in figure 2.

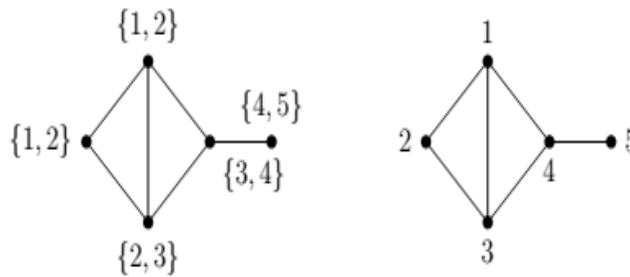


Figure 2: List Coloring

The figure here shows the color assignment without specification of initial colors. The first figure is showing the list of possible colors and second is showing the assigned colors. The color assignment and the list generation are here defined with specification of particular function. Once the list is generated, the next work is to apply the choosable function so that the final color assignment will be done. The constraint specific dual function mapping is defined to assign the effective colors to the graph.

B) Packed Coloring

Packed color is the resource driven analysis applied on the graph under the frequency observation. The basic concept of this coloring is to wave with the lower frequency having longer reach. The graph is here defined as the partitioned vertex set with disjoint set specification. Each of the partition is processed separately with class specification. As the color to the individual region is assigned it is considered as i-packing. The property driven distinct pairs are generated to assign the colors in effective way. The distance adaptive method is defined to select the color. The packed coloring is shown here in figure 3.

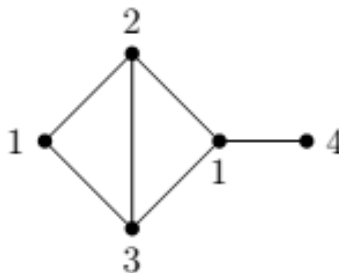


Figure 3: Packed Coloring

Here figure 3 is showing the packed coloring with 4 color assignment. The packed coloring initially begin from the empty set and later on apply the color assignment based on the eccentric coloring concept. The color assignment to a vertex cannot exceed the eccentricity of the node.

In this paper, the graph coloring exploration is provided in terms of characterization, behavior and the different colorization forms. In this section, the basic concept of graph coloring is defined along with constraint and representation methods specification. In section II, the related work on graph coloring provided by different researchers is presented. In section III, the graph coloring methods are defined. In section IV, the conclusion obtained from the work is presented.

II. RELATED WORK

Graph coloring is the common and bench mark algorithmic problem. The problem provides the solution to effective service allocation under constraint specification. In this section, the contributions of different researchers are provided to generate the optimized colorization solution. Author [8] has identified the common problems of coloring system and the graph theory. The K-color graph based processing was provided for two graphs and provided the sequential step and parallel step based evaluation. The node specific and the object specific measurements were provided by the author. Detailed description of author was provided for Asynchronous and Synchronous systems so that the effective k colors will be allocated over the graph. A trigger computation based membrane analysis was provided by the author. The evolution rules are also managed to generate the conditional specifications. The complexity driven estimation was provided by the author to achieve effective and adaptive color assignment. A study work on some of optimization methods in reference to the graph coloring problems was provided by Samar et. al. [17]. Author identified the major development and generated the algorithmic pertaining specific color assignment. The neural processing was defined by the author for rule formulation. The computational validity was provided by the author to achieve effective color assignment.

Alava et. al. [2] has defined a work on generalized distributed graph coloring to generate the local optimum solution. The edge weight based analysis was observed to identify the node color. The sub graph estimation was provided with maximum weight estimation so that the distributed colorization will be achieved. The local minima based performance evaluation was provided to perform effective color allocation. Author applied the greedy search algorithm for planner graph based colorization to achieve the effective weight driven estimation. Feizi et. al. [3] also characterizes the graph based on minimum entropy analysis. The functional analysis and computation is provided to generate the rate based region analysis to provide the solution in effective time. The graph colorization was provided to cover the connectivity observation of non-overlapping fully connected independent sets. The colorization solution was obtained within the polynomial time. The quantization solution was provided by the author to provide the colorization solution. Khot et. al. [5] has provided the constraint specific graph coloring using the approximation evaluation method. Author generated the adaptive graph features including Maximum Clique, Chromatic Number identification and finally mapping them to the graph nodes. The conceptual featured mapping is here provided to obtain the approximately connectivity and obtain the inherent color assignment in effective way. The complexity driven analysis and estimation was provided with linear constraint specification. Yumei et. al. [6] has applied the active branch analysis for applying the effective graph theory. Author defined the tournament search under frequency channel assignment, graph coloring and edge coloring in different graph regions. Torkestani et. al. [7] has defined automata learning method for effective graph color assignment. The minimum color utilization was provided to achieve the effective graph coloring with minimum run time. The size and the probability based parametric estimation was provided by the author to observe the node degree and the connectivity est. Author applied the legal coloring based on the maximum probability estimation. Another analytical and constraint specific work on graph coloring was provided by Doshi et. al. [9] using entropy based analysis. The graph characterization was provided by the author with functional encoding and correlation analysis. The preprocessing was provided by the author to generate the product graphs with modular technique specification. The product feature generalization and conditional feature map was provided to improve the information gain. The fidelity criteria specification based unified solution was generated by the author to generate the functional observation so that the graph color assignment will be done more adaptively. The relaxation rate and the distortion based functional analysis were provided by the author.

Tri et. al. [10] has used the constructive heuristic method for effective graph coloring in realistic graph environment. The method is here defined to generate the heuristic and the feasible solutions. The least saturation degree based graph coloring method was provided by the author. The insight solution was generated for constructive heuristic results are provided. The neighborhood functional analysis with solution matrix definition was provided to assign the particular color class to the nodes. The column specific heuristic feature map was provided for effective color assignment. Insani et. al. [11] has defined the data mining method for color assignment to graph nodes. The significant variance analysis with structural observational and graph color evaluation was provided by the author. The average instance based graph property analysis was provided to color the graph instances. Author defined a method for LP relaxation and defined the column generation for graph coloring. The Meta data processing and the heuristic search method were provided to generate the instance specific analysis. Graph feature analysis and the structural feature

processing were provided for adaptive color assignment. Another parameter considered for graph coloring was Energy [12]. Author defined the functional observation based on energy evaluation and energy consumption minimization. The circumvents for the energy consumption control was provided by the author so that the adaptive color assignment will be obtained from the work. The sub graph processing with k-coloring method was provided to improve the optimization method. The computational observation was provided to reduce the number of colors used and to provide the edge specific color observation. The quality driven estimation was provided to achieve the spanning graph generation. These induced sub graphs are here processed to apply the k coloring for the graph. Author defined the minimum energy function with neuron specific color assignment so that the rule specific graph processing will be done along with effective color utilization. The function driven regional estimation was provided with certain color count specification so that the effective graph scheduling and the colorization quality will be obtained. Another method with parallel graph computation was provided by the Yeh et. al. [13] using DNA based approach. The method adopted here is based on 3-coloring and k-coloring method and provided the positive instance analysis with molecular behavior observation. The sequence encoding for the strings is provided to assign the colors to nodes. Biological operation analysis was provided to assign the color in effective way. The adjacent node estimation and the connectivity observation were provided for graph color assignment. DNA sequence processing and the vertex specific estimation were provided for reducing the complexity. The complexity reduction was achieved by the author to obtain the effective feature processing. Author [15] defined the work on acyclic property of graph and provided the robust graph coloring. A sufficient condition was defined by the author to process the acyclic and k-partite feature of graph. The division specific method was provided with cubic planner graph to assign the colors in adaptive way. The subdivision specification and the coloring with partition processing were provided by the author.

Some of the researchers applied the optimization algorithms in integrated form to generate the effective colorization solution. Tomar et. al. [1] has used the ABC (Artificial Bee Colony) optimization algorithm to provide the graph coloring solution. The optimization was here achieved by utilizing the least number of chromatic numbers for graph. The colorization sequence and the degree adaptive ordering are defined to provide the best fit solution. The coverage adaptive solution was provided to assign the colors to all the graph vertices. Fister et. al. [4] has used the Differential Evolution method to provide the graph coloring optimization solution. The heuristic method was provided to generate the extensive work solution by processing the local search over graph. The interference analysis and connectivity analysis was provided to generate more accurate color assignment. Another work graph color assignment was provided using ACO [14] approach. Author defined the Meta heuristic method for color allocation and processing. The interference analysis and the relative ANT specific path generation were considered for processing the irregular network architectures. The phase specific analysis was provided to achieve the maximum convergence in interactive way. The complexity driven estimation and the directional observation was provided by the author. Author identified the corresponding solution in effective time and improves the solution quality. Another optimization to the graph coloring was provided by Raja et. al. [16] using genetic approach. The method employed here is based on the innovative single parent processing under crossover and mutation operators. The distance and the degree adaptive rules are applied as fitness rule for color evaluation and assignment. An edge triggered method was provided to improve the probabilistic estimation to generate the effective color assignment sequence. Gassen et. al. [18] used the neural network modeling for color minimization. The additional capability analysis with graph coloring was provided to maintain the convergence rate for color assignment in the robust graph environment.

III. GRAPH COLORING METHODS

Graph coloring is having the larger scope and the application area. Based on this, graph coloring is defined with different constraint and configuration setup. In this section, some of the common graph coloring forms used in different application areas are discussed with work behavior.

A) Four-Color Problem

In this problem form, the planner graphs are processed to color by using 4 colors. To assign the colors, new vertices and the connectivity can be added to the network. The vertex connectivity can be configured to apply the coloring with restricted number of colors. Another example of k-colorable graph is outer planner graph that can be colored by using maximum 3 colors. To perform the coloring some new vertex v can be added to the graph. Along with vertex, the connectivity nodes are also defined. The incident edges of that particular are identified to perform the colorization. One such outer planer graph is shown here in figure 4.

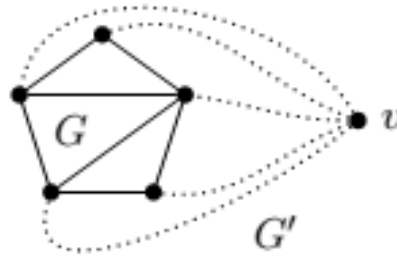


Figure 4: Outer planer Graph with Added Vertex v

Here figure 4 is showing the outer planer graph with specification of new vertex v. The graph can be coloring now by adding 3 colors.

B) Degeneracy

It is to perform the attack on graph. A graph is called k-generated graph, if each of its sub graph is having almost degree k. The overall degree of each node of graph G is almost k. Planner graphs are the example of k-generated graph with degree at most 5. Tree is also the Degeneracy graph with 1-degenerated and the outer planner are with 2-generated form. The degeneracy of the graph is effectively used to colorize the graph. Once the sub graphs with degree specification is obtained, the colorization theory can be applied to it. The effective region based color assignment can be applied. The induced sub graph based graph coloring can be applied for colorization.

C) Kempe Chain

Kempe Chains is the attempt discovered for effective graph coloring. The key observation for Kempe Chain is applied on proper colored graph. Let a sub graph H is extracted from graph with maximum connected 2-color. The colored graph was here generated by observing the proper coloring for

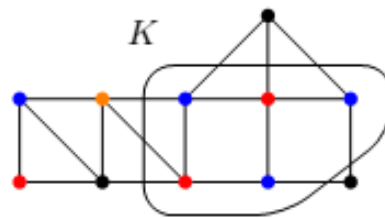


Figure 5: Kempe Chain based Graph Coloring

Each of the sub graphs. The maximal connected sub graph can be processed with specification of vertices and the available colors. An example of graph coloring using Kempe chain based planar graph is provided with 5-colorable. The Kempe based colorization is shown here in figure 5.

Here G is the sub graph with at most degree 4. The graph colorization is here defined using 5-colorable method. The neighbor specific analysis is applied on each of the sub graph to perform graph coloring. Kempe Chain method defined the number of assigned color and identifies the node degree. Later on it apply the color exchange and analyze the graph in sequential form. The extended colorization is here defined with graph color assignment. The switching of color is here done based on the neighborhood analysis. The color assignment is here done with specification of graph colorization. It is considered as the extended sequential colorization method defined to colorize the nodes with color specific evaluation.

D) Discharging

Discharging is the method applied to resolve the graph coloring problem for planner graphs. Discharging is the colorization process defined with specification of local constraints. The reducible configuration based evaluation is defined to perform effective colorization. At the earlier stage, the initial charges or cost is assigned to vertices and faces. The crucial property is here defined to obtain the aggregative cost of planner graph. The reducible configuration at vertex level is shown here figure 6

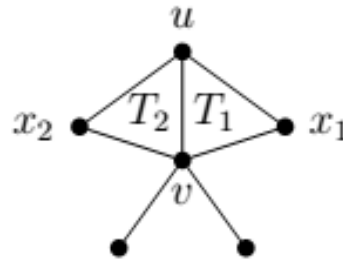


Figure 6: Reducible Graph Configuration

The reducible configuration is here defined based on the triangle specification. The sharing edge based configuration analysis is here provided. The vertex specific analysis is provided to generate the connected nodes. The minimum vertex based graph coloring is applied to perform effective colorization. The redistribute rule is here defined to provide the charges increase and provided the effective colorization. The adjacent node based analysis is here provided to provide the effective colorization.

IV. CONCLUSION

Graph coloring is the common and the multi-purpose problem used in different application under different aspects. The graph colorization is here provided for different methods, measures and constraint specification. The behavior specific methods are here defined to improve the colorization with minimum number of colors.

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