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Vendor-Buyer's Integrated Inventory Model with Quantity Discount, Delay in Payments and Advertisement Cost for Fixed Lifetime Products

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Abstract: Integrated inventory model have been derived for obtaining the economic order quantity for which the vendor permits a fixed delay in payments and trade credit to settling the accounts. One of the most important technologies in business field is advertisement. Usually in business environment, advertisement is a form of marketing can used to promote or sell something. During this coordination investigates advertisement expenditure and assume that shortages are not allowed and quantity discount is provided to the buyer. However, buyer makes the arrangement for screening or disposal of the damaged products. Mainly trade credit and delay in payments can be used to verify the quality of their products before having to pay for them. Furthermore, numerical examples are given to illustrate the results developed in this paper.

Keywords: Integrated Inventory, quantity discount, advertising, delay in payments, trade credit.

1. Introduction:

In today's global market, more companies realize that the performance of their business depends largely on advertising co-ordination. Advertisement plays an extremely important role in a nation's economy. The integrated inventory models mainly have the advantage of reducing total cost. A traditional EOQ model makes that the buyer needs to pay the full amount to the vendor when the products are received. But, in real market transactions, buyer do not need to pay the total amount at the time of product is received; they are allowed to delay in payments by the vendor. This type of trade credit is very common in today's business world.

By means of advertising, the vendor is known to offer for buyer a delay period for settling the payment for the goods and does not charge any interest from the buyer on the amount owned during this credit period. However, a higher interest is charged if the payment is not settled by the end of the credit period. The existence of the credit period serves to reduce the cost of holding stock to the buyer for reducing the amount of capital invested in stock. Advertising expenditures can be used to eliminate coordination failures, by allowing an efficient firm to communicate implicitly that it offers a lower price. During this coordination, shortages are not allowed, the quantity discount is provided to the buyer and the damaged products are not returned to the vendor. The buyer makes arrangement for screening or disposal of the damaged products and also assumes limited lifetime for each item.

The main aim of this paper is to study an integrated inventory model with advertisement, quantity discount, delay in payments and trade credit. The remainder of this paper is organized as follows, section 2 presents a review of related literature, section 3 defines assumptions and notations, section 4 formulate the mathematical model, section 5 provides some numerical example and finally we conclude the paper in section 6.

2. Literature Review:

Several researchers have developed analytical inventory models with consideration of permissible delay in payments. Goyal (1985) established a single-item inventory model under permissible delay in payments. Yung-Fu Huang (2013) classified an EOQ model under supplier trade credit policy depending on the order quantity. Chung (1998) developed an alternative approach to determine the economic order quantity under condition of permissible delay in payments. Hung- Chi Chang (2011) studied an economic order quantity with imperfect quality and quantity discounts. Chen and Kang (2010) analyzed coordination between vendor and buyer considering trade credit and items of imperfect quality. Liu and Shi (1999) classified perishability and deteriorating inventory models into two major categories namely decay models and finite lifetime models. Duan et al. (2010) developed buyer-vendor inventory coordination with quantity discount incentive for fixed lifetime product. They mainly addressed buyer-vendor coordination by quantity discount. P.Muniappan, R.Uthayakumar and S.Ganesh (2015) classified a production inventory model for vendor-buyer coordination with quantity discount, backordering and rework for fixed life time products. Huang, Li(2002),Zhu and Chau (2002) developed independently a co-op advertising model for a one manufacturer, one retailer supply chain and Huang and Li (2001) in developing a price discount model to coordinate advertising expanses of two parties.

3. Assumptions and Notations:

The following assumptions and notations are used to develop the model.

Notations:

D	Annual demand
P	Production rate per year
L	Lifetime of product
$k_{1,}k_2$	Vendor and buyer's setup costs per order respectively
$h_{1,}h_2$	Vendor and buyer's holding costs respectively
$p_{1,}p_2$	Delivered unit price paid by the vendor and the buyer respectively
Q	Order quantity
C_{d1}	Vendor's unit production cost
C_2	Buyer's unit purchase cost
v_s	Vendor's unit screening cost
n	Vendor's order multiple under coordination
c_s	Unit disposal cost for scrap items
K	Buyer's order multiple under coordination
$d(K)$	Denotes the per unit dollar discount to the buyer
I	Initialization cost

f_1 Fixed cost per advertisement

V_1 Variable cost per advertisement

n_1 Number of times per advertisement taken

t Cast to telecasting the advertisement

W Minimum order quantity at which the delay in payments is permitted

c Unit purchasing price per item

s Unit selling price per item

M The trade credit period

T The cycle time

I_e Interest which can be earned per year

I_p Interest charges per investment in inventory per year

B Backordering ratio

Assumptions:

1) Demand is known and constant.

2) Shortages are not allowed.

3) Time period is infinite.

4) The buyer does not return the damaged products instead make arrangement for screening or disposed for damaged products.

5) If $Q < W,$ i.e, $T < W/D$, the delayed payment is not permitted. Otherwise, fixed trade credit M is permitted. Hence, if $Q < W$, pay cQ when the order is received. If $Q \geq W,$ pay cQM time periods after the order is received.

6) During the time the account is not settled, generated sales revenue is deposited in an interest-bearing account. When$T \geq M$, the account is settled at $T = M$, the buyer pays off all units sold and keeps profits and starts paying for the higher interest charges on the items in stock. When$T \leq M$, the account is settled at $T = M$ and the buyer does not need to pay any interest charge.

7) $s \geq c, I_p \geq I_e$

4. Mathematical Model

We derive the integrated vendor-buyer's total cost is formulated as follows

TC = Buyer's ordering cost + Buyer's holding cost + Buyer's screening cost + Buyer's disposal cost + Vendor's ordering cost + Vendor's holding cost + The buyer's quantity discount given by vendor + advertisement cost + Interest Payable − Interest earned.

$$TC = \frac{Dk_2}{Q} + \frac{h_2(1-B)^2QC_2}{2} + \frac{V_sQ}{2} + \frac{c_suvQ}{2} + \frac{Dk_1}{nKQ} + \frac{h_1KQc_{d1}}{2}\left[(n-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right] + p_2Dd(k) + \frac{[I+f_1+V_1+n_1t]}{T} + \text{Interest payable} - \text{Interest earned.}$$

There are three cases to occur in cost of interest charges for the items kept in stock per year.

Case (i): $0 < T < W/D$

Cost of interest charges per year = $\frac{cI_pDT}{2}$

Cost of interest earned per year is zero.

Case (ii): $W/D \leq T \leq M$

In this case, No interest charges paid for the items.

Cost of interest earned per year = $DsI_e\left(M-\frac{T}{2}\right)$

Case (iii): $M \leq T$

Cost of interest charges per year = $\frac{cI_pD(T-M)^2}{2T}$

Cost of interest earned per year = $\frac{DM^2sI_e}{2T}$

We formulate the total cost for these three cases as follows:

Case (i): $0 < T < W/D$

$$TC_1 = \frac{Dk_2}{Q} + \frac{h_2(1-B)^2QC_2}{2} + \frac{v_sQ}{2} + \frac{C_suvQ}{2} + \frac{Dk_1}{nKQ}$$

$$+ \frac{h_1KQc_{d1}}{2}\left[(n-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right] + p_2Dd(k) + \frac{[I+f_1+V_1+n_1t]}{T} + \frac{cI_pDT}{2}$$

Now, we have to find the optimum of$Q$, that is

$$\frac{dTC_1}{dQ} = 0$$

i.e, $-\frac{Dk_2}{Q^2} + \frac{h_2(1-B)^2C_2}{2} + \frac{V_s}{2} + \frac{c_suv}{2} - \frac{Dk_1}{nKQ^2}$
$+ \frac{h_1Kc_{d1}}{2}\left[(n-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right] - \frac{[I+f_1+V_1+n_1t]D}{Q^2} + \frac{cI_p}{2} = 0$

We get,

$$Q^* = \sqrt{\frac{2D\left[\left(\frac{k_1}{nK}+k_2\right)+(I+f_1+v_1+n_1t)\right]}{h_2(1-B)^2C_2+v_s+c_suv+cI_p+\frac{h_1KC_{d1}}{2}\left[(n-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right]}}$$

To find the optimum of B, that is

$$\frac{dTC_1}{dB} = \frac{h_2 2BQC_2}{2} - \frac{2h_2QC_2}{2} = 0$$

It follows that, $B^* = 1$

Similarly, to find the optimum of T, that is

$$T^* = \sqrt{\frac{2\left[\left(\frac{k_1}{nK}+k_2\right)+(I+f_1+v_1+n_1t)\right]}{D\left(h_2(1-B)^2C_2+v_s+c_suv+cI_p+h_1KC_{d1}(n-1)-\frac{h_1KC_{d1}D}{P}(n-2)\right)}}$$

Under the coordination the problem can be used as such that

$$\begin{cases} nKt_0 \leq L \\ \frac{Dk_2}{KQ} + \frac{h_2(1-B)^2KQC_2}{2} + \frac{v_sKQ}{2} + \frac{C_suvKQ}{2} + \frac{[I+f_1+V_1+n_1t]D}{KQ} + \frac{cI_pKQ}{2} \\ \quad - \sqrt{\frac{2D\left[\left(\frac{k_1}{nK}+k_2\right)+(I+f_1+v_1+n_1t)\right]}{h_2(1-B)^2C_2+v_s+c_suv+cI_p+h_1KC_{d1}\left[(n-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right]}} \leq p_2Dd(k) \\ n \geq 1 \end{cases} \quad \ldots\ldots(1)$$

If second constraint of (1) is an equation that

$$d(k) = \frac{\frac{Dk_2}{KQ} + \frac{h_2(1-B)^2KQC_2}{2} + \frac{v_sKQ}{2} + \frac{C_suvKQ}{2} + \frac{[I+f_1+V_1+n_1t]D}{KQ} + \frac{cI_pKQ}{2} - \sqrt{\frac{2D\left[\left(\frac{k_1}{nK}+k_2\right)+(I+f_1+v_1+n_1t)\right]}{h_2(1-B)^2C_2+v_s+c_suv+cI_p+h_1KC_{d1}\left[(n-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right]}}}{p_2D}$$

Case (ii): $W/D \leq T \leq M$

$$TC_2 = \frac{Dk_2}{Q} + \frac{h_2(1-B)^2QC_2}{2} + \frac{v_sQ}{2} + \frac{C_suvQ}{2} + \frac{Dk_1}{nKQ} + \frac{h_1KQC_{d1}}{2}\left[(n-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right] + p_2Dd(k) + \frac{[I+f_1+V_1+n_1t]}{T} - DsI_e\left(M-\frac{T}{2}\right)$$

Now, we have to find the optimum of Q,

$$\frac{dTC_2}{dQ} = \frac{-D}{Q^2}\left[k_2 + \frac{k_1}{nK} + ([I+f_1+V_1+n_1t])\right] + \frac{1}{2}\left[h_2(1-B)^2QC_2 + v_s + c_suv + h_1Kc_{d1}\left[(n-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right] + sI_e\right] = 0$$

Therefore,

$$Q^* = \sqrt{\frac{2D\left[\left(\frac{k_1}{nK}+k_2\right)+(I+f_1+v_1+n_1t)\right]}{h_2(1-B)^2C_2+v_s+c_suv+sI_e+h_1KC_{d1}\left[(n-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right]}}$$

Similarly, to find the optimum of T, that is

$$T^* = \sqrt{\frac{2\left[\left(\frac{k_1}{nK}+k_2\right)+(I+f_1+v_1+n_1t)\right]}{D\left(h_2(1-B)^2C_2+v_s+c_suv+sI_e+h_1KC_{d1}(n-1)-\frac{h_1KC_{d1}D}{P}(n-2)\right)}}$$

Similarly, to find the optimum value of B,

We get, $B^* = 1$

Under the coordination the problem can be used as such that

$$\begin{cases} nKt_0 \le L \\ \frac{Dk_2}{KQ} + \frac{h_2(1-B)^2KQC_2}{2} + \frac{v_sKQ}{2} + \frac{C_suvKQ}{2} + \frac{[I+f_1+V_1+n_1t]D}{KQ} + \frac{sI_eQK}{2} - DsI_eMK \\ \quad - \sqrt{\frac{2D\left[\left(\frac{k_1}{nK}+k_2\right)+(I+f_1+v_1+n_1t)\right]}{h_2(1-B)^2C_2+v_s+c_suv+sI_e+h_1KC_{d1}\left[(n-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right]}} \le p_2Dd(k) \\ n \ge 1 \end{cases}$$

……………. (2)

If second constraint of (2) is an equation that

$$d(k) = \frac{\frac{Dk_2}{KQ} + \frac{h_2(1-B)^2KQC_2}{2} + \frac{v_sKQ}{2} + \frac{C_suvKQ}{2} + \frac{[I+f_1+V_1+n_1t]D}{KQ} + \frac{sI_eQK}{2} - DsI_eMK - \sqrt{\frac{2D\left[\left(\frac{k_1}{nK}+k_2\right)+(I+f_1+v_1+n_1t)\right]}{h_2(1-B)^2C_2+v_s+c_suv+sI_e+h_1KC_{d1}\left[(n-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right]}}}{p_2D}$$

Case (iii): $M \le T$

$$TC_3 = \frac{Dk_2}{Q} + \frac{h_2(1-B)^2QC_2}{2} + \frac{v_sQ}{2} + \frac{C_suvQ}{2} + \frac{Dk_1}{nKQ} + \frac{h_1KQc_{d1}}{2}\left[(n-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right] + p_2Dd(k) + \frac{[I+f_1+V_1+n_1t]}{T} + \frac{cI_pD(T-M)^2}{2T} - \frac{DM^2sI_e}{2T}$$

Now, we have to find the optimum of Q,

$$\frac{dTC_2}{dQ} = -\frac{1}{2Q^2}\left[2Dk_2 + \frac{2Dk_1}{nK} + 2[I + f_1 + V_1 + n_1 t]D + cI_p D^2M^2 - D^2M^2 sI_e\right] + \frac{1}{2}\left[h_2(1-B)^2C_2 + v_s + c_s uv + h_1Kc_{d1}\left[(n-1)\left(1-\frac{D}{P}\right) + \frac{D}{P}\right] + cI_p\right] = 0$$

Therefore,

$$Q^* = \sqrt{\frac{2D\left[\left(k_2+\frac{k_1}{nK}\right)+(I+f_1+v_1+n_1 t)\right]+D^2M^2(cI_p-sI_e)}{h_2(1-B)^2C_2+v_s+c_s uv+h_1KC_{d1}\left[(n-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right]+cI_p}}$$

Similarly, to find the optimum of T, that is

$$T^* = \sqrt{\frac{2\left[\left(\frac{k_1}{nK}+k_2\right)+(I+f_1+v_1+n_1 t)+\frac{DM^2}{2}(cI_p-sI_e)\right]}{D\left(h_2(1-B)^2C_2+v_s+c_s uv+cI_p+h_1KC_{d1}(n-1)-\frac{h_1KC_{d1}D}{P}(n-2)\right)}}$$

Similarly, to find the optimum value of B,

That is, $B^* = 1$

Under the coordination the problem can be used as such that

$$\begin{cases} nKt_0 \le L \\ \frac{Dk_2}{KQ} + \frac{h_2(1-B)^2KQC_2}{2} + \frac{v_sKQ}{2} + \frac{C_s uvKQ}{2} + \frac{[I+f_1+V_1+n_1 t]D}{KQ} + \frac{cI_pQK}{2} + \frac{D^2M^2}{2QK}\left(cI_p - sI_e\right) - cI_pDMK \\ \quad - \sqrt{\frac{2D\left[\left(\frac{k_1}{nK}+k_2\right)+(I+f_1+v_1+n_1 t)\right]+D^2M^2(cI_p-sI_e)}{h_2(1-B)^2C_2+v_s+c_s uv+cI_p+h_1KC_{d1}\left[(n-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right]}} \le p_2Dd(k) \\ n \ge 1 \end{cases}$$

$$\ldots\ldots\ldots\ldots.(3)$$

If second constraint of (3) is an equation that

$$d(k) = \frac{\frac{Dk_2}{KQ} + \frac{h_2(1-B)^2KQC_2}{2} + \frac{v_SKQ}{2} + \frac{C_SuvKQ}{2} + \frac{[I+f_1+V_1+n_1t]D}{KQ} + \frac{cI_pQK}{2} + \frac{D^2M^2}{2QK}(cI_p - sI_e) - cI_pDMK - \sqrt{\frac{2D\left[\left(\frac{k_1}{nK}+k_2\right)+(I+f_1+v_1+n_1t)\right]+D^2M^2(cI_p-sI_e)}{h_2(1-B)^2C_2+v_S+c_Suv+cI_p+h_1KC_{d1}\left[(n-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right]}}}{p_2D}$$

5. Numerical Example:

To illustrate the usefulness of the model developed in section 4, let us consider the following parameter for analyzing the above inventory is given below:
$P = 2000$ Units/year; $D = 1000$ units/year; $p_2 = \$5$;$L = 0.25$;
$k_1 = 300$Per order;$k_1 = \$100$;$h_1 = 10\$$; $c_{d1} = 3\$$ per order; $C_2 = 4$;
$v_s = 2\$$Per order;$u = 0.20$;$v = 0.15$;$c_s = 1\$$per order;$c = 30\$$ per unit;
$I_p = 0.15\$/\text{Year}$;$I_e = 0.1\$/\text{year}$;$s = 60\$/\text{unit}$;$M = 0.1\text{year}$;$n = 1$;$K = 1$
The computational results shows the optimal values

Case (i): $0 < T < W/D$
$Q^* = 877.37053$;$B^* = 1$;$T^* = 0.87737$;$d(k) = 1.560014$;$TC_1 = 20{,}109.581$

Case (ii): $W/D \leq T \leq M$
$Q^* = 833.9234$;$B^* = 1$;$T^* = 0.83392$;$d(k) = 1.605990$;$TC_1 = 20{,}380.794$

Case (iii): $M \leq T$
$Q^* = 876.76103$;$B^* = 1$;$T^* = 0.87676$;$d(k) = 1.46883$;$TC_1 = 19{,}195.11165$

6. Conclusion:

This paper represents a vendor-buyer's integrated inventory model with quantity discount, delay in payments and advertisement cost for fixed life time products. From the numerical example, we have cleared that integrated inventory is more beneficial and we find that the result of less order quantity to take the gain of the delay payment.

References:

1. Chen, L. H. and F. S. Kang, "Coordination between vendor and buyer considering trade credit and items of imperfect quality," International Journal of Production Economics, 123, 52–61 (2010).

2. K. J. Chung (1998), A theorem on the determination of economic order quantity under conditions of permissible delay in payments, Computers and Operations Research, Vol. 25, pp. 49–52.

3. Fries, B. "Optimal order policies for a perishable commodity with fixed life time," Operations Research, 23, 46–61 (1975).

4. Fujiwara, O., H. Soewandi and D. Sedarage, "An optimal and issuing policy for a two-stage inventory system for perishable products," European Journal of Operational Research, 99, 412–424 (1997).

5. S. K. Goyal (1985), Economic order quantity under conditions of permissible delay in payments, Journal of the Operational Research Society, Vol. 36, pp. 35–38.

6. S. K Goyal(1989). and Y. P. Gupta, "Integrated inventory models: The buyer–vendor coordination," European Journal of Operational Research, 41, 261–269.

7. Hung - Chi Chang, "A comprehensive note on: An economic order quantity with imperfect quality and quantity discounts," Applied Mathematical Modelling, 35, 5208–5216 (2011).

8. Kanchana, K. and T. Anulark, "An approximate periodic model for fixed-life perishable products in a two-echelon inventory distribution system," International Journal of Production Economics, 100, 101–115 (2006).

9. Lian, Z. and L. Liu, "Continuous review perishable inventory systems: Models and heuristics," IIE Transactions, 33, 809–822 (2011).

10. Liu, L. and Z. Lian, "(s, S) model for inventory with fixed life time," Operations Research, 47, 130–158 (1999).

11. Liu, L. and D. Shi, "(s, S) model for inventory with exponential life times and renewal demands," Naval Research Logistics, 46, 39–56 (1999).

12. Duan, Yongrui, Jianwen Luo and Jiazhen Huo, "Buyer– vendor inventory coordination with quantity discount incentive for fixed lifetime product," International Journal of Production Economics, 128, 351–357 (2010).

13. Jaber, Mohamad Y. and Ahmed M. A. El Saadany, "An economic production and remanufacturing model with learning effects," International Journal of Production Economics, 131, 115–127 (2011).

14. P.Muniappan, R.Uthayakumar and S.Ganesh, "A production inventory model for vendor–buyer coordination with quantity discount, backordering and rework for fixed life time products," Journal of Industrial and Production and Engineering, 1-8 (2015).

15. Kreng, Victor B. and Shao-Jung Tan, "Optimal replenishment decision in an EPQ model with defective items under supply chain trade credit policy," Expert Systems with Applications, 38, 9888–9899 (2011).

16. Wahab, M. I. M. and M. Y. Jaber, “Economic order quantity model for items with imperfect quality, different holding costs, and learning effects: A note,” Computers and Industrial Engineering, 58, 186–190 (2010).
17. Wong, W. K., J. Qi and S. Y. S Leung, “Coordinating supply chains with sales rebate contracts and vendor- managed inventory,” International Journal of Production Economics, 120, 151–161 (2009).
18. Wahab, M. I. M. and M. Y. Jaber, “Economic order quantity model for items with imperfect quality, different holding costs, and learning effects: A note,” Computers and Industrial Engineering, 58, 186–190 (2010).
19. Yung-Fu Huang, “An EOQ model under supplier trade credit policy depending on the order quantity,” Journal of Information and Optimization Sciences, 26:2, 311-326 (2005).
20. Yuan-Shyi Peter Chiu, Shang-Chih Liu, Chun-Lin Chiu and Huei-Hsin Chang, “Mathematical modelling for determining the replenishment policy for EMQ model with rework and multiple shipments,” Mathematical and Computer Modelling, 54, 2165–2174 (2011).

