Inpainting using Multiresolution Gaussian Pyramid

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Abstract—Image inpainting is the technique of filling-in damaged regions by the surrounding information without affecting the contents and quality of the image i.e. restoration of an image using computers. Bertalmio proposed the first digital inpainting technique and coined the word ‘inpainting’ to the restoration work of digital images. Digital image inpainting is an interesting research topic and gathered the much needed momentum since 2000. In the present paper, we generate multiresolution Gaussian pyramid of an image to be inpainted by filtering and down sampling. We substitute the damaged region of an image at a next lowest resolution level by expanding image at the lowest resolution level. The expansion and substitution process is carried out from the lowest resolution to the highest resolution to fill-in the damaged region. The downsampling reduces the damaged regions along the Gaussian pyramid and the expansion (upsampling) and substitution fills-in the damaged region.

Keywords—Inpainting, Gaussian pyramid, up sampling, down sampling.

I. INTRODUCTION

The concept of image inpainting was introduced by Bertalmio-Sapiro-Caselles-Ballester [2]. Since then, there is a wide interest in this area as it finds many applications starting from image restoration to image compression. In the conventional form of inpainting, user selects areas for inpainting and the algorithm automatically fills-in these regions with information surrounding them.

In this paper, we restrict our interest to structure inpainting technique. Input image $f_0$ is down sampled using Gaussian pyramid up to $L$ levels to obtain $f_{-1}, f_{-2}, f_{-3}, \ldots \ldots f_{-L}$, images at levels $2^{-1}$ to $2^{-L}$, respectively. The inpainting region $\Omega$ is reduced by down sampling. After $2^{-L}$ level, expand the image $f_{-L}$ to obtain $f'_{-L+1}$, substitute damaged region in image $f'_{-L+1}$ by corresponding region from $f_{-L+1}$. This process is repeated from highest level (lowest resolution) to lowest level (original resolution) to obtain inpainted image output. The results are good for both small and medium size inpainting areas.
II. RELATED WORK

The inpainting techniques can be classified into two broad categories: Structure and Texture inpainting.

II.1 STRUCTURE INPAINTING

In structure inpainting the damaged region is filled by diffusing neighboring information of image. Bertalmio [2] was the first person to propose digital inpainting in the year 2000. He proposed an inpainting technique algorithm based on the PDE. This method is an extension of level lines based on disocclusion method proposed by Masnou and J-M Morel [1]. After user selects the region to be inpainted, this method iteratively propagates the information from the outside of the inpainting area to inside of the inpainting area (diffusion) along the isophote lines. The direction of propagation of information is obtained by computing a discretized gradient vector and rotating this vector by 90 degree. Oliveira[3] proposed a fast image inpainting method which depends on the convolution operation. The algorithm uses a weighted average kernel that has a zero weight at the center of the kernel. The algorithm is fast and works well for images which do not have many contrast edges or high frequency components (e.g. natural texture, edges).

In [4], Handhoud presents a modification of Oliveira’s algorithm with reduced time complexity. H. Noori [5] proposed a method in which mask coefficients are calculated using the gradient of the image to be inpainted.

Roth and Black [6] have developed Field of Expert (FoE) framework to learn generic and expressive image priors that captures the statistics of natural scenes. The approach extends traditional Markov Random Field (MRF) models by learning potential functions over the extended pixel neighborhoods. Field potentials are modeled using a Products-of-Experts framework that exploits nonlinear functions of many linear filter responses. Authors demonstrated the capabilities of Field of Experts (FoE) model with two examples i.e. image denoising and image inpainting. In [7] a bilateral kernel used for convolution is obtained by multiplying range and space kernels. For each pixel, kernels are calculated using its neighbors in space and range domains. Since the bilateral filters are efficient in denoising, inpainting can be performed by estimating the lost (damaged) pixels.

II.2 TEXTURE INPAINTING

Texture inpainting is relatively difficult since texture is to be either synthesized or pasted from surrounding area of damaged region based on some similarity properties. The statistical properties of texture play an important role to synthesize and measure similarity between two textures. The damaged or missing regions are filled in using similar neighborhood texture in the image. The statistics of boundary of damaged regions should be similar to statistics of known neighborhood regions. Hirani and Totsuka [11] combine frequency and spatial domain information in order to fill a given region with a selected texture. Other texture synthesis algorithms, e.g., [12,13] can be used as well to recreate a preselected texture to fill in a square region to be inpainted. The algorithm mainly deals with texture synthesis and not with structured background. In [14], Efros proposed a nonparametric texture synthesis model based Markov Random Field to inpaint textural images. In his method, first a neighborhood around a damaged pixel is selected and then the known regions of image are searched to find the most similar region to the selected neighborhood. Finally, the central pixel found in neighborhood is copied to the damaged pixel. This method is time consuming and it does not produce good results in structured regions.

Criminisi [15] modified model in [14] to achieve better results. Chang [16] showed that priority function adopted in [14] is not a well defined function and may become unreliable after a number of iteration. Therefore they proposed a new function to assign priority to pixels providing a robust exemplar based algorithm. Bertalmio [17] proposed to decompose original image into textural and structural sub images. The structural sub image is reconstructed by a structure algorithm and the textural sub image is restored by a texture synthesis approach. Then, two processed components are combined to obtain inpainted image. Similar approach is proposed in [18], in which instead of decomposing the image, the original image is segmented into two sub regions. Rajkumar [19] proposed texture inpainting based second order statistical parameters like correlation and covariance which are used to determine similar patches in the image.

All above approaches work better when region to be inpainted remains small. This paper proposes a faster method to inpaint larger areas using multiresolution Gaussian pyramid.
III. MULTiresolution APPROXIMATION OF AN IMAGE

The multiresolution approximation of one dimensional signal \( f(x) \) can be easily extended to two dimensional signal, image. The discrete approximation of an image \( f(x, y) \in L^2(\mathbb{R}^2) \) at resolution \( 2^j \) is defined as

\[
A_j f = \left( f * \theta \left( 2^{-j} m, 2^{-j} n \right) \right)_{(m,n)\in\mathbb{Z}^2}
\]  

(6)

where \( \theta(x, y) \) is a two dimensional low pass filter, and \( \theta_j(x, y) = 2^j \theta(2^j x, 2^j y) \). The low pass filter \( \theta(x, y) \) is chosen such that its Fourier transform can be written as

\[
\hat{\theta}(\omega_x, \omega_y) = \prod_{n=1}^{\infty} \mathcal{H}(e^{-2\pi \cdot \omega_x}) \mathcal{H}(e^{-2\pi \cdot \omega_y})
\]  

(7)

Figure 1: Gaussian pyramid: The original image is approximated at resolutions 1/2, 1/4 and 1/8. As resolution decreases the details are lost and image is characterized by fewer pixels.

Implementation

Consider an image \( f_{i,u}(u, v) \), \( 1 \leq u \leq M, 1 \leq v \leq N \) at resolution \( 2^{i+1} \), \( -L \leq l \leq -1 \). The image \( f_j(u, v) \) at resolution \( 2^j \) which is reduced by half in both resolution and sample density is given by

\[
f_j(u, v) = \sum_{m=2u-2}^{2u} \sum_{n=2v-2}^{2v} f_{i+1}(2u + m, 2v + n)w(m, n)
\]

(8)

The expanded version of an image \( f_j(u, v) \) at resolution \( 2^j \) i.e. the image \( f_{i,u}(u, v) \) at resolution \( 2^{i+1} \) is given by

\[
f_{i+1}(u, v) = \sum_{m=2u-2}^{2u} \sum_{n=2v-2}^{2v} f_i \left( \frac{u-m}{2}, \frac{v-n}{2} \right)w(m, n)
\]

(9)

where \( w(m, n) \) is Gaussian filter of size \( 5 \times 5 \) and is used to generate Gaussian pyramid. This window is called as Gaussian generating kernel [17]. The window \( w(m, n) \) is separable i.e. \( w(m, n) = w(m)w(n) \). where \( w(\cdot) \) is a normalized one dimension Gaussian filter of length 5 and is given by

\[
\sum_{m=2}^{2} w(m) = 1
\]

(10)

And it is symmetric: \( w(-i) = w(i) \) for \( i = 0, 1, 2 \) and having equal contribution. This stipulates that all pixels at a given level contribute the same total weight to the pixels at the next lower resolution level. Let \( w(0) = a, w(-1) = w(1) = b \) and \( w(-2) = w(2) = c \) then equal contribution requires that \( a + 2c = 2b \). These three constraints are satisfied when

\[
w(0) = a
\]

\[
w(-1) = w(1) = 1/4
\]

\[
w(-2) = w(2) = 1/4 - a/2
\]

(11)
The \( w(m) \) is unimodal such that \( w(i) > w(j) \) for \( 0 < i < j \), therefore \( a > 0.25 \), \( b = 0.25 \) and \( c = 0.25 - a/2 \). Here the parameter \( a \) of the generating kernel is chosen to be 0.4 such that the resulting Gaussian kernel resembles the Gaussian probability density function with zero mean and unity variance. The shapes of Gaussian kernel for different values of \( a \) are shown in the Figure 1. for \( a = 0.6 \) the central positive mode is sharply peaked and is flanked by small negative lobes, for \( a = 0.5 \) the shape is triangular and for \( a = 0.3 \) it is flatter and broader. Thus for \( a = 0.4 \) the one dimension Gaussian kernel is \( w(m) = [0.05 \ 0.25 \ 0.4 \ 0.25 \ 0.05] \). This kernel can also be obtained by discretizing zero mean unity variance Gaussian probability density function.

\[
\begin{bmatrix}
0.0025 & 0.0125 & 0.0200 & 0.0125 & 0.0025 \\
0.0125 & 0.0625 & 0.1000 & 0.0625 & 0.0125 \\
0.0200 & 0.1000 & 0.1600 & 0.1000 & 0.0200 \\
0.0125 & 0.0625 & 0.1000 & 0.0625 & 0.0125 \\
0.0025 & 0.0125 & 0.0200 & 0.0125 & 0.0025
\end{bmatrix}
\]

(12)

Figure 2: Shapes of Gaussian generating kernels for different values of \( a \).

The Gaussian kernels being separable, the 2D Gaussian kernel is given by

\[
w(m, n) = w'(m)w(m)
\]

IV. INPAINTING USING MULTiresolution GAUSSIAN PYRAMID

Generate all the possible routes. In this technique, a Gaussian pyramid [17] of resolutions \( 2^{-1} \) to \( 2^{-L} \) of an input image is generated. The input image \( f_0 \) is downsampled using Gaussian pyramid up to \( L \) resolution levels to obtain images \( f_{-1}, f_{-2}, \ldots, f_{-L} \) at resolution levels \( 2^{-1} \) to \( 2^{-L} \), respectively. The damaged region \( \Omega_0 \) progressively reduces to \( \Omega_l = \Omega_0 2^l, -L \leq l \leq -1 \). After \( 2^{-L} \) resolution levels the expand operation is applied. The image \( f_{-L} \) corresponding to lowest resolution is expanded to obtain \( f_{-L+1} \).
The damaged region, $\Omega$ of image $f_L$ is considered and the pixels of this region are substituted by the pixels of corresponding region from $f'_{L-1}$. This substitution of Gaussian interpolated pixels assumes the processes of diffusion at that resolution. The expand operation followed by substitution is repeated for subsequent resolution levels up to original resolution to obtain inpainted image as shown in the inpainting algorithm. Figure 3 shows the inpainting procedure discussed for a typical image. The Gaussian pyramid for $L=3$ is constructed. The left top image is the image to be inpainted and the left bottom image is the resulting inpainted image.

The Inpainting Algorithm

$f_0(u,v) :$ Input image with $\Omega_0$ as damaged region to be filled-in and at the end of algorithm input image is replaced by inpainted image.

$L: $ Maximum number of resolution levels.

$f_i(u,v) :$ Image at resolution $2^i$ of Gaussian pyramid with $\Omega_i$ as inpainting region to be filled at $2^i$ resolution.

$w(m,n) :$ Two dimension Gaussian kernel.

Reduce operation:

for $(l = -1 : -1 : -L)$

$$f_l(u,v) = \sum_{m=2n=2}^{L} f_{l+1}(2u + m, 2v + n)w(m,n)$$

Store $[f_i(u,v), \Omega_i]$

End

$f_{-L}(u,v)$ is the output image of the reduce operation with reduced damaged region $\Omega_{-L}$, where $1 \leq u \leq M / 2^l$ and $1 \leq v \leq N / 2^l$.

Expand and Substitution operation:

for $(l = -L : -1 : -1)$

$$f'_{l+1}(u,v) = \sum_{m=2n=2}^{2} f_{l+1}(\frac{u-m}{2}, \frac{v-n}{2})w(m,n)$$

$$f_{l+1}(u,v) = f'_{l+1}(u,v) \text{ for } \forall (u,v) \in \Omega_{l+1}$$

End

Expand operation results output image $f_0(u,v)$ after an iteration replacing the original input image. For larger damaged regions above Reduce followed by Expand and Substitution operations are repeated more than once.
V. IMAGE SET AND PERFORMANCE MEASURE

A wide variety of images is used to investigate the performance of proposed inpainting technique. The prior knowledge of the source, type or direction of lighting or camera parameters is not important in the present context. The techniques are independent of lighting and camera parameters and are not considered. Hence, images are collected from different sources viz. standard damaged images used by an image processing researchers (available on internet), synthetic images, biomedical image, amateur photographs taken by the author etc.

To test performance of the proposed technique, clean undamaged images are artificially degraded, and then inpainted. Although, the evaluation of inpainting technique/s is to be assessed by visual appearance and quality which is subjective in nature, a quantitative assessment would be more objective. There is no established quantitative measure for this evaluation. Hence in our work, the quantitative evaluation is performed by peak signal to noise ratio (PSNR). The PSNR is defined as-

\[
PSNR = 20 \log_{10} \left( \frac{255}{RMSE(f_{clean}, f_{inp})} \right)
\]

where \(RMSE(f_{clean}, f_{inp}) = \sqrt{ \frac{ \sum_{u,v} (f_{clean}(u,v) - f_{inp}(u,v))^2 }{MN} }\) \(,\) (14)

where \(f_{clean}(u,v)\) is clean undamaged image, \(f_{inp}(u,v)\) is inpainted image and \(M \times N\) is size of \(f_{clean}(u,v)\) and \(f_{inp}(u,v)\). Please note that this measure is applicable only when the undamaged original image is available.

VI. RESULTS

For every image to be inpainted, a binary mask image corresponding to inpainting area is created manually as shown in Figures (4c-7c). The white lines in the mask image correspond to damaged region and are the region of an interest (not shown). To establish the efficacy of our techniques, we artificially degrade the clean undamaged images by drawing random lines (artificial scratches), or pasting new objects on the images to create damaged image (artificial stains). These damaged images are inpainted (restored) and compared with the clean undamaged original images to calculate PSNR. Figures (4a-7a) show clean undamaged images. The scratches are created randomly to degrade the images as shown in Figures (4b-7b). These scratches are inpainted using Gaussian pyramid inpainting technique as shown in Figures (4f to 7f). The comparison of proposed technique is carried out by using the codes written by Bertalmio’s [2] (Figures (4d-7d)) in C++ and Stefan Roth [6] (Figures (4e-7e)) in MATLAB without any major changes. However their PSNR values and execution time is tabulated in Table 1 in comparison with proposed technique.

Figure 4 Inpainting of artificial lines on Lena’s image. (a): Clean image. (b): Damaged by putting artificial lines. (c): Mask image. (d): Bertalmio’s technique, PSNR= 49.765 dB and Exe. time=128.580 S. (e): FoE technique, PSNR= 45.6642 dB and Exe. time = 445.947 S. (f): Gaussian pyramid technique, PSNR= 50.6885dB and Exe. time = 2.625 S.
Figure 5 Inpainting of artificial lines and scratches on Cameraman’s image. (a): Clean image. (b): Damaged image. (c): Mask image. (d): Bertalmio’s technique, PSNR= 41.217 dB and Exe. time=135.386 S. (e): FoE technique with PSNR= 38.505 dB & Exe. time= 1119.111 S. (f): Gaussian pyramid technique, PSNR= 43.285 dB & Exe. time= 4.450 S.

Figure 6 Inpainting of artificial lines on fruits image. (a): Clean image. (b): Damaged by putting artificial lines. (c): Mask image. (d): Bertalmio’s technique, PSNR= 38.489 dB and Exe. time=506.567 S. (d): FoE technique, PSNR= 34.514 dB and Exe. time= 2591.503 S. (e): Gaussian pyramid technique, PSNR= 42.199 dB and Exe. time= 2.2911 S.
Figure 7 Inpainting of artificial scratches on group photograph. (a): Clean photograph. (b): Damaged photo. (c): Mask image. (d): Bertalmio’s technique, PSNR= 39.103 dB and Exec. time= 5.032 S. (e): FoE technique, PSNR= 40.181 dB and Exec. time= 309.030 S. (f): Gaussian pyramid technique with PSNR= 40.103 dB and Exec. time= 3.754 S.

The PSNR values and inpainting time taken by each technique for each image is tabulated in Table 1. We observe that PSNR measure is in tandem with the visual quality i.e. better the PSNR and better is the inpainted image. The comparison of Gaussian pyramid technique is carried out with the techniques of Bertalmio’s and Stefan Roth (FoE). The proposed method shows the better performance in terms of PSNR, visual quality and execution time. By the observation of Table 1, we can say that Inpainting using multiresolution Gaussian pyramid is a fast technique and takes appreciably less time than that of Bertalmio’s and FoE techniques.

### VII. Conclusion and Future Work

The proposed algorithm is fast and produces good inpainting results. The results are better in terms of both quantitative assessment and visual quality than some of the previous methods [2, 6] proposed. For very large the result is not encouraging, as expected & the proposed technique only work for structure.

### REFERENCES


