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RESEARCH ARTICLE

The Differential Problem of Two Types of Functions

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Abstract—This paper takes the mathematical software Maple as the auxiliary tool to study the differential problem of two types of functions. We can obtain the infinite series forms of any order derivatives of these two types of functions by using binomial series and differentiation term by term theorem. The research methods adopted in this study involved finding solutions through manual calculations and verifying these solutions by using Maple. This type of research method not only allows the discovery of calculation errors, but also helps modify the original directions of thinking from manual and Maple calculations. For this reason, Maple provides insights and guidance regarding problem-solving methods.

Keywords—derivatives, infinite series forms, binomial series, differentiation term by term theorem, Maple

I. Introduction

As information technology advances, whether computers can become comparable with human brains to perform abstract tasks, such as abstract art similar to the paintings of Picasso and musical compositions similar to those of Mozart, is a natural question. Currently, this appears unattainable. In addition, whether computers can solve abstract and difficult mathematical problems and develop abstract mathematical theories such as those of mathematicians also appears unfeasible. Nevertheless, in seeking for alternatives, we can study what assistance mathematical software can provide. This study introduces how to conduct mathematical research using the mathematical software Maple. The main reasons of using Maple in this study are its simple instructions and ease of use, which enable beginners to learn the operating techniques in a short period. By employing the powerful computing capabilities of Maple, difficult problems can be easily solved. Even when Maple cannot determine the solution, problem-solving hints can be identified and inferred from the approximate values calculated and solutions to similar problems, as determined by Maple. For this reason, Maple can provide insights into scientific research. Inquiring through an online support system provided by Maple or browsing the Maple website (www.maplesoft.com) can facilitate further understanding of Maple and might provide unexpected insights. For the instructions and operations of Maple, we can refer to [1]-[7].

In calculus and engineering mathematics courses, determining the n-th order derivative value $f^{(n)}(c)$ of a function $f(x)$ at $x=c$, in general, needs to go through two procedures: firstly finding the n-th order derivative $f^{(n)}(x)$ of $f(x)$, and secondly taking $x=c$ into $f^{(n)}(x)$. These two procedures will make us face with increasingly complex calculations when calculating higher order derivative values of a function (i.e. n is large), Therefore, to obtain the answers by manual calculations is not easy. In this paper, we mainly study the differential problem of the following two types of functions

$$f(x) = x^r (a+bx^p)^q \tag{1}$$

$$g(x) = e^{rx} (a+be^{px})^q \tag{2}$$

Where a, b, p, q, r are real numbers, and $p > 0$, $a, b \neq 0$, a^q, b^q exist. We can obtain the infinite series forms of any order derivatives of these two types of functions by using binomial series and differentiation term by term theorem; these are the major results of this study (i.e., Theorems 1, 2), and hence greatly reduce the difficulty of evaluating the higher order derivative values of these two types of functions. As for the related study of differential problems can refer to [8]-[16]. On the other hand, we propose some examples to do calculation practically. The research methods adopted in this study involved finding solutions through manual calculations and verifying these solutions by using Maple. This type of research method not only allows the discovery of calculation errors, but also helps modify the original directions of thinking from manual and Maple calculations. For this reason, Maple provides insights and guidance regarding problem-solving methods.

II. MAIN RESULTS

Firstly, we introduce a notation and two important theorems used in this study.

Notation. Suppose r is any real number, m is any positive integer. Define $(r)_m = r(r-1)\cdots(r-m+1)$, and $(r)_0 = 1$.

Binomial series ([17]).

If w, a are real numbers, $|w| < 1$. Then $(1+w)^a = \sum_{k=0}^{\infty} \frac{(a)_k}{k!} w^k$.

Differentiation term by term theorem ([18]).

If, for all non-negative integer k , the functions $g_k : (a,b) \to R$ satisfy the following three conditions : (i) there exists a point $x_0 \in (a,b)$ such that $\sum_{k=0}^{\infty} g_k(x_0)$ is convergent, (ii) all functions $g_k(x)$ are differentiable on open interval (a,b) , (iii) $\sum_{k=0}^{\infty} \frac{d}{dx} g_k(x)$ is uniformly convergent on (a,b) . Then $\sum_{k=0}^{\infty} g_k(x)$ is uniformly convergent and differentiable on (a,b) . Moreover, its derivative $\frac{d}{dx} \sum_{k=0}^{\infty} g_k(x) = \sum_{k=0}^{\infty} \frac{d}{dx} g_k(x)$.

The following is the first result in this study, we obtain the infinite series forms of any order derivatives of function (1).

Theorem 1. Assume a, b, p, q, r are real numbers, n is any positive integer, and $p > 0$, $a, b \neq 0$, a^q, b^q exist. Let the domain of the function $f(x) = x^r (a + bx^p)^q$ be $\left\{ x \in R \,\middle|\, x^r (a+bx^p)^q, x^{pq} \, exist, x \neq 0, \pm \left|\frac{a}{b}\right|^{1/p} \right\}$.

Case (a): If $|x| < \left|\frac{a}{b}\right|^{1/p}$ and $x \neq 0$, then the n -th order derivative of $f(x)$,

$$f^{(n)}(x) = a^q \cdot \sum_{k=0}^{\infty} \frac{(q)_k (pk+r)_n}{k!} \left(\frac{b}{a}\right)^k x^{pk+r-n} \tag{3}$$

Case (b): If $|x| > \left|\frac{a}{b}\right|^{1/p}$, then

$$f^{(n)}(x) = b^q \cdot \sum_{k=0}^{\infty} \frac{(q)_k (-pk+pq+r)_n}{k!} \left(\frac{a}{b}\right)^k x^{-pk+pq+r-n} \tag{4}$$

Proof. Case (a) : If $|x| < \left|\frac{a}{b}\right|^{1/p}$, then $\left|\frac{b}{a} x^p\right| < 1$. Therefore,

$$f(x) = x^r (a + bx^p)^q$$

$$= a^q x^r \left(1 + \frac{b}{a} x^p\right)^q$$

$$= a^q x^r \cdot \sum_{k=0}^{\infty} \frac{(q)_k}{k!} \left(\frac{b}{a} x^p\right)^k \quad \text{(using binomial series)}$$

$$= a^q \cdot \sum_{k=0}^{\infty} \frac{(q)_k}{k!} \left(\frac{b}{a}\right)^k x^{pk+r} \tag{5}$$

By differentiation term by term theorem, differentiating n-times with respect to x on both sides of (5), we obtain the n-th order derivative of $f(x)$,

$$f^{(n)}(x) = a^q \cdot \sum_{k=0}^{\infty} \frac{(q)_k (pk+r)_n}{k!} \left(\frac{b}{a}\right)^k x^{pk+r-n}$$

for all x satisfy $|x| < \left|\frac{a}{b}\right|^{1/p}$ and $x \neq 0$.

Case (b): If $|x| > \left|\frac{a}{b}\right|^{1/p}$, then $\left|\frac{a}{b} x^{-p}\right| < 1$. Thus

$$f(x) = x^r (a + bx^p)^q$$

$$= b^q x^{pq} \cdot x^r \left(1 + \frac{a}{b} x^{-p}\right)^q$$

$$= b^q x^{pq+r} \cdot \sum_{k=0}^{\infty} \frac{(q)_k}{k!} \left(\frac{a}{b} x^{-p}\right)^k \quad \text{(using binomial series)}$$

$$= b^q \cdot \sum_{k=0}^{\infty} \frac{(q)_k}{k!} \left(\frac{a}{b}\right)^k x^{-pk+pq+r} \tag{6}$$

Also, using differentiation term by term theorem, differentiating n-times with respect to x on both sides of (6), we determine the n-th order derivative of $f(x)$,

$$f^{(n)}(x) = b^q \cdot \sum_{k=0}^{\infty} \frac{(q)_k (-pk+pq+r)_n}{k!} \left(\frac{a}{b}\right)^k x^{-pk+pq+r-n}$$

for all x satisfy $|x| > \left|\frac{a}{b}\right|^{1/p}$ ∎

Next, we determine the infinite series forms of any order derivatives of function (2).

Theorem 2. Let the assumptions be the same as Theorem 1, and suppose the domain of the function $g(x) = e^{rx}(a + be^{px})^q$ is $\left\{ x \in R \,\middle|\, (a + be^{px})^q \, exist, e^x \neq \left|\frac{a}{b}\right|^{1/p} \right\}$.

Case (a): If $e^x < \left|\frac{a}{b}\right|^{1/p}$, then the n-th order derivative of $g(x)$,

$$g^{(n)}(x) = a^q \cdot \sum_{k=0}^{\infty} \frac{(q)_k (pk+r)^n}{k!} \left(\frac{b}{a}\right)^k e^{(pk+r)x} \tag{7}$$

Case (b): If $e^x > \left|\frac{a}{b}\right|^{1/p}$, then

$$g^{(n)}(x) = b^q \cdot \sum_{k=0}^{\infty} \frac{(q)_k(-pk+pq+r)^n}{k!}\left(\frac{a}{b}\right)^k e^{(-pk+pq+r)x} \tag{8}$$

Proof. Case (a) : If $e^x < \left|\frac{a}{b}\right|^{1/p}$, then $\left|\frac{b}{a}e^{px}\right| < 1$. Hence,

$$\begin{aligned} g(x) &= e^{rx}(a+be^{px})^q \\ &= a^q e^{rx}\left(1+\frac{b}{a}e^{px}\right)^q \\ &= a^q e^{rx} \cdot \sum_{k=0}^{\infty}\frac{(q)_k}{k!}\left(\frac{b}{a}e^{px}\right)^k \quad \text{(using binomial series)} \\ &= a^q \cdot \sum_{k=0}^{\infty}\frac{(q)_k}{k!}\left(\frac{b}{a}\right)^k e^{(pk+r)x} \end{aligned} \tag{9}$$

By differentiation term by term theorem, differentiating n-times with respect to x on both sides of (9), we obtain the n-th order derivative of $g(x)$,

$$g^{(n)}(x) = a^q \cdot \sum_{k=0}^{\infty} \frac{(q)_k(pk+r)^n}{k!}\left(\frac{b}{a}\right)^k e^{(pk+r)x}$$

for all x satisfy $e^x < \left|\frac{a}{b}\right|^{1/p}$.

Case (b): If $e^x > \left|\frac{a}{b}\right|^{1/p}$, then $\left|\frac{a}{b}e^{-px}\right| < 1$. Therefore

$$\begin{aligned} g(x) &= e^{rx}(a+be^{px})^q \\ &= b^q e^{pqx} \cdot e^{rx}\left(1+\frac{a}{b}e^{-px}\right)^q \\ &= b^q e^{(pq+r)x} \cdot \sum_{k=0}^{\infty}\frac{(q)_k}{k!}\left(\frac{a}{b}e^{-px}\right)^k \quad \text{(using binomial series)} \\ &= b^q \cdot \sum_{k=0}^{\infty}\frac{(q)_k}{k!}\left(\frac{a}{b}\right)^k e^{(-pk+pq+r)x} \end{aligned} \tag{10}$$

Also using differentiation term by term theorem, differentiating n-times with respect to x on both sides of (10), we determine the n-th order derivative of $g(x)$,

$$g^{(n)}(x) = b^q \cdot \sum_{k=0}^{\infty} \frac{(q)_k(-pk+pq+r)^n}{k!}\left(\frac{a}{b}\right)^k e^{(-pk+pq+r)x}$$

for all x satisfy $e^x > \left|\frac{a}{b}\right|^{1/p}$ ∎

III. EXAMPLES

Regarding the differential problem of the two types of functions in this study, we provide some examples and use Theorems 1, 2 to determine the infinite series forms of their any order derivatives. In addition, we evaluate some higher order derivative values of these functions and employ Maple to calculate the approximations of these higher order derivative values and their infinite series forms for verifying our answers.

Example 1. Suppose the domain of the function

$$f_1(x) = \sqrt[5]{x^4} \cdot \left(45 - 2 \cdot \sqrt[3]{x^{11}}\right)^4 \tag{11}$$

is $\left\{ x \in R \middle| x \neq 0, \pm\left(\frac{45}{2}\right)^{3/11} \right\}$ (the case of $a = 45, b = -2, r = \frac{4}{5}, p = \frac{11}{3}, q = 4$ in Theorem 1).

Case (i): If $|x| < \left(\frac{45}{2}\right)^{3/11} \cong 2.3376$ and $x \neq 0$, then by Case (a) of Theorem 1, we obtain the infinite series form of any n-th order derivative of $f_1(x)$,

$$f_1^{(n)}(x) = 45^4 \cdot \sum_{k=0}^{\infty} \frac{(4)_k (11k/3 + 4/5)_n}{k!} \left(\frac{-2}{45}\right)^k x^{11k/3+4/5-n} \tag{12}$$

Therefore, we can determine the 4-th order derivative value of $f_1(x)$ at $x = 2$,

$$f_1^{(4)}(2) = 45^4 \cdot \sum_{k=0}^{\infty} \frac{(4)_k (11k/3 + 4/5)_4}{k!} \left(\frac{-2}{45}\right)^k 2^{11k/3-16/5} \tag{13}$$

In the following, we use Maple to verify the correctness of (13).

```
>f1:=x->x^(4/5)*(45-2*x^(11/3))^4;
```

$$f1 := x \to x^{4/5} \left(45 - 2\, x^{11/3}\right)^4$$

```
>evalf((D@@4)(f1)(2),20);
```

$$-3.2472031168503943930 \cdot 10^8$$

```
>evalf(45^4*sum(product(4-j,j=0..(k-1))*product(11*k/3+4/5-i,i=0..3)/k!*(-2/45)^k*2^(11*k/3-16/5),k=0..
infinity),20);
```

$$-3.247203116850394394 \cdot 10^8$$

Case (ii): If $|x| > \left(\frac{45}{2}\right)^{3/11} \cong 2.3376$, then by Case (b) of Theorem 1, we obtain the infinite series form of any n-th order derivative of $f_1(x)$,

$$f_1^{(n)}(x) = (-2)^4 \cdot \sum_{k=0}^{\infty} \frac{(4)_k (-11k/3 + 232/15)_n}{k!} \left(-\frac{45}{2}\right)^k x^{-11k/3+232/15-n} \tag{14}$$

Hence, we can evaluate the 7-th order derivative value of $f_1(x)$ at $x = 6$,

$$f_1^{(7)}(6) = (-2)^4 \cdot \sum_{k=0}^{\infty} \frac{(4)_k (-11k/3 + 232/15)_7}{k!} \left(-\frac{45}{2}\right)^k 6^{-11k/3+127/15} \tag{15}$$

In the following, we use Maple to verify the correctness of (15).

```
>evalf((D@@7)(f1)(6),20);
```

$$2.6201453169023157288 \cdot 10^{15}$$

```
>evalf((-2)^4*sum(product(4-j,j=0..(k-1))*product(-11*k/3+232/15-i,i=0..6)/k!*(-45/2)^k*6^(-11*k/3+127/15)
,k=0..infinity),20);
```

$$2.6201453169023157288 \cdot 10^{15}$$

Example 2. Let the domain of the function

$$f_2(x) = \frac{\sqrt[3]{x^7}}{\left(9 + 4 \cdot \sqrt[11]{x^5}\right)^2} \tag{16}$$

be $\left\{ x \in R \middle| x \neq 0, \pm\left(\frac{9}{4}\right)^{11/5} \right\}$ (the case of $a = 9, b = 4, r = \frac{7}{3}, p = \frac{5}{11}, q = -2$ in Theorem 1).

Case (i): If $|x| < \left(\frac{9}{4}\right)^{11/5} \cong 5.9539$ and $x \neq 0$, then by Case (a) of Theorem 1, we obtain the infinite series form of any n -th order derivative of $f_2(x)$,

$$f_2^{(n)}(x) = 9^{-2} \cdot \sum_{k=0}^{\infty} \frac{(-2)_k (5k/11 + 7/3)_n}{k!} \left(\frac{4}{9}\right)^k x^{5k/11+7/3-n} \tag{17}$$

Thus, we can determine the 6-th order derivative value of $f_2(x)$ at $x = 5$,

$$f_2^{(6)}(5) = 9^{-2} \cdot \sum_{k=0}^{\infty} \frac{(-2)_k (5k/11 + 7/3)_6}{k!} \left(\frac{4}{9}\right)^k 5^{5k/11-11/3} \tag{18}$$

In the following, we use Maple to verify the correctness of (18).

```
>f2:=x->x^(7/3)*(9+4*x^(5/11))^(-2);
```

$$f2 := x \to \frac{x^{7/3}}{\left(9 + 4\, x^{5/11}\right)^2}$$

```
>evalf((D@@6)(f2)(5),14);
```

0.000010419017603695

```
>evalf(9^(-2)*sum(product(-2-j,j=0..(k-1))*product(5*k/11+7/3-i,i=0..5)/k!*(4/9)^k*5^(5*k/11-11/3),k=0..
infinity),20);
```

0.0000104190176036924

Case (ii): If $|x| > \left(\frac{9}{4}\right)^{11/5} \cong 5.9539$, by Case (b) of Theorem 1, we obtain the infinite series form of any n -th order derivative of $f_2(x)$,

$$f_2^{(n)}(x) = 4^{-2} \cdot \sum_{k=0}^{\infty} \frac{(-2)_k (-5k/11 + 47/33)_n}{k!} \left(\frac{9}{4}\right)^k x^{-5k/11+47/33-n} \tag{19}$$

Therefore, we can evaluate the 8-th order derivative value of $f_2(x)$ at $x = 12$,

$$f_2^{(8)}(12) = 4^{-2} \cdot \sum_{k=0}^{\infty} \frac{(-2)_k (-5k/11 + 47/33)_8}{k!} \left(\frac{9}{4}\right)^k 12^{-5k/11-217/33} \tag{20}$$

We also use Maple to verify the correctness of (20).

```
>evalf((D@@8)(f2)(12),24);
```

$$7.61340851042919753243284 \cdot 10^{-8}$$

>evalf(4^(-2)*sum(product(-2-j,j=0..(k-1))*product(-5*k/11+47/33-i,i=0..7)/k!*(9/4)^k*12^(-5*k/11-217/33), k=0..infinity),28);

$$7.613408510429197532431 5 \cdot 10^{-8}$$

Example 3. If the domain of the function

$$g_1(x) = e^{3x/2} \cdot \sqrt[5]{(8+3e^{4x/7})^9} \tag{21}$$

is $\left\{ x \in R \middle| e^x \neq \left(\frac{8}{3}\right)^{7/4} \right\}$ (the case of $a = 8, b = 3, r = \frac{3}{2}, p = \frac{4}{7}, q = \frac{9}{5}$ in Theorem 2).

Case (i): If $e^x < \left(\frac{8}{3}\right)^{7/4}$, i.e., $x < \frac{7}{4}\ln\left(\frac{8}{3}\right) \cong 1.7164$. Then by Case (a) of Theorem 2, we obtain the infinite series form of any n-th order derivative of $g_1(x)$,

$$g_1^{(n)}(x) = 8^{9/5} \cdot \sum_{k=0}^{\infty} \frac{(9/5)_k (4k/7+3/2)^n}{k!} \left(\frac{3}{8}\right)^k e^{(4k/7+3/2)x} \tag{22}$$

Thus, we can determine the 11-th order derivative value of $g_1(x)$ at $x = 4/3$,

$$g_1^{(11)}(4/3) = 8^{9/5} \cdot \sum_{k=0}^{\infty} \frac{(9/5)_k (4k/7+3/2)^{11}}{k!} \left(\frac{3}{8}\right)^k e^{16k/21+2} \tag{23}$$

Using Maple to verify the correctness of (23) as follows:

>g1:=x->exp(3*x/2)*(8+3*exp(4*x/7))^(9/5);

$$g1 := x \rightarrow e^{\frac{3}{2}x} \left(8 + 3\, e^{\frac{4}{7}x}\right)^{9/5}$$

>evalf((D@@11)(g1)(4/3),14);

$$6.4529772703051 \cdot 10^{6}$$

>evalf(8^(9/5)*sum(product(9/5-j,j=0..(k-1))*(4*k/7+3/2)^11/k!*(3/8)^k*exp(16*k/21+2),k=0..infinity),14);

$$6.4529772703056 \cdot 10^{6}$$

Case (ii): If $e^x > \left(\frac{8}{3}\right)^{7/4}$, i.e., $x > \frac{7}{4}\ln\left(\frac{8}{3}\right) \cong 1.7164$, by Case (b) of Theorem 2, we can evaluate the infinite series form of any n-th order derivative of $g_1(x)$,

$$g_1^{(n)}(x) = 3^{9/5} \cdot \sum_{k=0}^{\infty} \frac{(9/5)_k (-4k/7+177/70)^n}{k!} \left(\frac{8}{3}\right)^k e^{(-4k/7+177/70)x} \tag{24}$$

Therefore, we obtain the 12-th order derivative value of $g_1(x)$ at $x = 7$,

$$g_1^{(12)}(7) = 3^{9/5} \cdot \sum_{k=0}^{\infty} \frac{(9/5)_k (-4k/7+177/70)^{12}}{k!} \left(\frac{8}{3}\right)^k e^{-4k+177/10} \tag{25}$$

We also use Maple to verify the correctness of (25).

>evalf((D@@12)(g1)(7),24);

$$2.41043574021050027981418 \cdot 10^{13}$$

>evalf(3^(9/5)*sum(product(9/5-j,j=0..(k-1))*(-4*k/7+177/70)^12/k!*(8/3)^k*exp(-4*k+177/10),k=0..infinity),24);

$$2.41043574021050027981416\cdot 10^{13}$$

Example 4. Suppose the domain of the function

$$g_2(x) = \frac{1}{e^{5x/7}(4+5e^{2x/9})^{3/8}} \tag{26}$$

is $\left\{x \in R \middle| e^x \neq \left(\frac{4}{5}\right)^{9/2}\right\}$ (the case of $a=4, b=5, r=-\frac{5}{7}, p=\frac{2}{9}, q=-\frac{3}{8}$ in Theorem 2).

Case (i): If $e^x < \left(\frac{4}{5}\right)^{9/2}$, i.e., $x < \frac{9}{2}\ln\left(\frac{4}{5}\right) \cong -1.0041$. Then by Case (a) of Theorem 2, we obtain the infinite series form of any n-th order derivative of $g_2(x)$,

$$g_2^{(n)}(x) = 4^{-3/8}\cdot\sum_{k=0}^{\infty}\frac{(-3/8)_k(2k/9-5/7)^n}{k!}\left(\frac{5}{4}\right)^k e^{(2k/9-5/7)x} \tag{27}$$

Thus, we can evaluate the 4-th order derivative value of $g_2(x)$ at $x=-6/7$,

$$g_2^{(4)}(-6/7) = 4^{-3/8}\cdot\sum_{k=0}^{\infty}\frac{(-3/8)_k(2k/9-5/7)^4}{k!}\left(\frac{5}{4}\right)^k e^{-4k/21+30/49} \tag{28}$$

In the following, we use Maple to verify the correctness of (28).

>g2:=x->1/(exp(5*x/7)*(4+5*exp(2*x/9))^(3/8));

$$g2 := x \rightarrow \frac{1}{e^{\frac{5}{7}x}\left(4+5\,e^{\frac{2}{9}x}\right)^{3/8}}$$

>evalf((D@@4)(g2)(-6/7),14);

0.26222723931592

>evalf(4^(-3/8)*sum(product(-3/8-j,j=0..(k-1))*(2*k/9-5/7)^4/k!*(5/4)^k*exp(-4*k/21+30/49),k=0..infinity),14);

0.26222723931592

Case (ii): If $e^x > \left(\frac{4}{5}\right)^{9/2}$, i.e., $x > \frac{9}{2}\ln\left(\frac{4}{5}\right) \cong -1.0041$, by Case (b) of Theorem 2, we can determine the infinite series form of any n-th order derivative of $g_2(x)$,

$$g_2^{(n)}(x) = 5^{-3/8}\cdot\sum_{k=0}^{\infty}\frac{(-3/8)_k(-2k/9-67/84)^n}{k!}\left(\frac{4}{5}\right)^k e^{(-2k/9-67/84)x} \tag{29}$$

Thus, we obtain the 5-th order derivative value of $g_2(x)$ at $x=3$,

$$g_2^{(5)}(3) = 5^{-3/8}\cdot\sum_{k=0}^{\infty}\frac{(-3/8)_k(-2k/9-67/84)^5}{k!}\left(\frac{4}{5}\right)^k e^{-2k/3-67/28} \tag{30}$$

Using Maple to verify the correctness of (30) as follows:

>evalf((D@@5)(g2)(3),18);

$$-0.0112962331485403841$$

```
>evalf(5^(-3/8)*sum(product(-3/8-j,j=0..(k-1))*(-2*k/9-67/84)^5/k!*(4/5)^k*exp(-2*k/3-67/28),k=0..
infinity),18);
```

$$-0.0112962331485403841$$

IV. CONCLUSIONS

As mentioned, the binomial series and the differentiation term by term theorem play significant roles in the theoretical inferences of this study. In fact, the applications of these two theorems are extensive, and can be used to easily solve many difficult problems; we endeavor to conduct further studies on related applications.

On the other hand, Maple also plays a vital assistive role in problem-solving. In the future, we will extend the research topic to other calculus and engineering mathematics problems and solve these problems by using Maple. These results will be used as teaching materials for Maple on education and research to enhance the connotations of calculus and engineering mathematics.

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