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RESEARCH ARTICLE

MEMORY EFFICIENT WDR (WAVELET DIFFERENCE REDUCTION) using INVERSE OF ECHELON FORM by EQUATION SOLVING

Dr. Sandeep Dalal[1], Poonam Rani[2]

[1]Computer Science and application DCSA,
Maharshi Dayanand University, Rohtak, Assistant Professor,
[2]Computer Science and application DCSA,
Maharshi Dayanand University, Rohtak, M.Tech Scholar,
[1] sandeepdalal.80@gmail.com; [2] poonam41188@gmail.com

Abstract— This is an attempt to devise a memory efficient WDR (Wavelet Difference Reduction) algorithm by decomposing a matrix using customized Echelon algorithm and applying WDR on it in parts. The standard process of WDR algorithm requires the entire matrix to be available in RAM (random access memory) which might not be feasible always or for a longer duration till the entire matrix is encoded. This idea takes advantage of working on matrix in parts without having to partition the matrix in any predefined way. The encoded matrix can be reconstructed using symbolic computations by creating equations and solving them against decomposed matrices.

Keywords— WDR, Row Reduction, Inverse of Row reduction, Matrix, Echelon

I. INTRODUCTION

Beauty of WDR (Wavelet Difference Reduction) algorithm is encoding the positions of significant bits rather than encoding the significant bits themselves. It also provides control to stop the encoding process once the needed characteristics are achieved. It allows working on region of interest. Its simplicity lies in the fact that it converts 2D matrix into 1D array before encoding, while other algorithms on similar lines, do work on data structures that employ matrices without any transformation, for example SPIHT makes use of Spatial orientation tree. In WDR, the encoding process could be stopped at any moment when some target quality has been met. Wavelet means small wave, wavelet transform is capable of transferring the signal into time and frequency domain simultaneously. Wavelet analysis can be used to divide the information of a 2D signal into approximation and detail sub-band. Detail sub band further represents the information of horizontal, vertical and diagonal information of an image or data. In this paper use the concept of row reduction or echelon form.

Definition Reduced Row Echelon Form. A matrix is said to be in row-reduced echelon form provided that [1].

- In each row that does not consist of all zero elements, the first non-zero element in this row is a 1. (Called. a "leading 1).
- In each column that contains a leading 1 of some row, all other elements of this column are zeros.
- In any two successive rows with non-zero elements, the leading 1 of the lower row occurs farther to the right than the leading 1 of the higher row.
- If there are any rows contain only zero elements then they are grouped together at the bottom.

II. ORIGINAL WDR(WAVELET DIFFERENCE REDUCTION) ALGORITHM

WDR algorithm consists of five parts shown in fig 1. First in the Initialize part, during this step an assignment of a scan order should be made first. Scan order should be created on the basis of wavelet coefficients. The row based scanning is used in the horizontal sub band, column based scanning is used in the vertical sub band, and zigzag scanning is used for the diagonal and low pass sub band as the scanning order is made . An initial threshold value T_0 is chosen so that all transformed values have magnitude less than T_0 and at least one has magnitude greater than or equal to$T_0/2$. The purpose of is to encode significant transform values by the method of bit-plane encoding. A binary expansion, relative to the quantity T_0 is computed for each transform value. The loop constitutes the procedure by which these binary expansions are calculated. Second in the update threshold, the threshold is successively reduced by half.

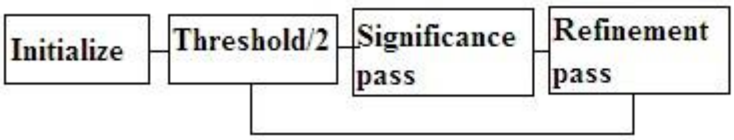

Fig 1 WDR algorithm

Third the significance pass, each transformed value is checked against the threshold value, if it is greater than the threshold than it is significant otherwise insignificant. Then significant index values are encoded using the difference reduction method of Tian and Wells [4]. The difference reduction method essentially consists of a binary encoding of the number of steps to go from the index of the last significant value to the index of the current significant value. The output from the significance pass includes the signs of significant value along with sequence of bits generated by difference reduction which describes the precise locations of significant values. Fourth is refinement pass, which is to generate the refined bits via the standard bit-plane quantization procedure. Each refined value is a better approximation of an exact transform value. Last step is to repeat steps second to fourth until the bit budget is reached[3][4][5][9].

A. WDR(Wavelet Difference Reduction) Algorithm

The WDR algorithm is very simple. Take any image or matrix data and perform wavelet transform and carried out the wavelet coefficients. WDR mainly consists of five steps as follows [5]:

1. (Initialize). Choose an initial threshold T_0 so that all transform value s satisfy $|X_m| < T_0$ and at least one transform value satisfies $|X_m| \geq T_0/2$.

2. (Update threshold). Let $T_K = T_{k-1}/2$.

3. (Significance Pass). Perform the following procedure while scanning through insignificant values for higher thresholds:
   Initialize step-counter $C = 0$
   Let $C_{old} = 0$
   Do
   - Get next insignificant index m
   - Increment step-counter C by 1
   - If $|X_m| \geq T_k$ then
     - Output $sign(X_m)$ and set $q_m = sign(X_m).T_k$
     - Move m to end of sequence of significant indices
     - Let $n = C - C_{old}$
     - Set $C_{old} = C$
     - If $n > 1$ then
       - Output reduced binary Expansion of n
     - Else if $|X_m| < T_k$ then
       - Let q_m retain its initial value of 0
   - Loop until end of insignificant indices out end-marker.

   The end–marker is a plus sign followed by the reduced binary expansion of $n = C + 1 - C_{old}$ and a final plus sign.

4. (Refinement pass). Scan through significant value found with higher threshold values T_j, for $j < k$(if $k = 1$ skip this step) for each significant value X_m do the following:
   - If $|X_m| \epsilon (|q_m|, |q_m| + T_k)$ then
     - Output bit 0
   - Else if $|X_m| \epsilon (|q_m| + T_k , |q_m| + 2T_k)$ then
     - Output bit 1
   - Repeat value of q_m by $q_m = sign(X_m .T_k)$

5. (Loop). Repeat step 2 through 4 (exit at any point if bit budget is exceeded).

III. METHOD USED TO MODIFY WDR ALGORITHM

The following method is improvised to modify the WDR algorithm.

A. Row Reduction Algorithm

There are couple of algorithms available to decompose a matrix into such matrices whose product will regenerate the original matrix. For example LU factorization, QR decomposition, singular value decomposition etc. Step of row reduction algorithm [6].

- Begin with the leftmost nonzero column. The pivot position is at the top.
- Select a nonzero entry in the pivot column as a pivot. If necessary, interchange rows to move this entry into the pivot position.
- Use row addition operations to create zeros in all position below the pivot.
- Cover (or ignore) the row containing the pivot position and cover all rows, if any, above it. Apply step 1-3 to the sub-matrix that remains. Repeat the process until there are no more non-zeros rows to modify.

Taking a simple matrix

$$\begin{pmatrix} 9 & 19 & 27 & 9 \\ 4 & 22 & 20 & 6 \\ 15 & 26 & 24 & 12 \\ 29 & 21 & 11 & 23 \end{pmatrix}$$

On being row reduced generates

$$\begin{pmatrix} 1.0 & 2.1111 & 3.0 & 1.0 \\ 0.0 & 1.0 & 0.59016 & 0.14754 \\ 0.0 & 0.0 & 1.0 & 0.1225 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{pmatrix}$$

B. Symbolic Representation of Row Reduction Algorithm

To get an overview of how elements are getting manipulated during the process of row reduction, symbolic computation can prove to be of great help and is the basis for motivation of this paper.

Sample 4*4 matrix,

$$\begin{pmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{pmatrix}$$

On applying row reduction generates

$$\begin{pmatrix} 1 & \frac{b}{a} & \frac{c}{a} & \frac{d}{a} \\ 0 & 1 & \frac{g-\frac{ce}{a}}{f-\frac{be}{a}} & \frac{h-\frac{de}{a}}{f-\frac{be}{a}} \\ 0 & 0 & 1 & \frac{l-\frac{\left(h-\frac{de}{a}\right)(j-\frac{bi}{a})}{(f-\frac{be}{a})}-\frac{di}{a}}{k-\frac{\left(g-\frac{ce}{a}\right)(j-\frac{bi}{a})}{f-\frac{be}{a}}-\frac{ci}{a}} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

This un-simplified version helps understand the reuse of computations done in early steps. On being simplified and reduced further,

$$\begin{pmatrix} 1 & \frac{b}{a} & \frac{c}{a} & \frac{d}{a} \\ 0 & 1 & \frac{ag-ce}{af-be} & \frac{ab-de}{af-be} \\ 0 & 0 & 1 & \frac{(af-be)l+(de-ah)j+(bh-df)i}{(af-be)k+(ce-ag)j+(bg-cf)i} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

IV. PROPOSED ALGORITHM

Extracting information from echelon method: this is a proposal to break down a matrix in sub-components in such a way that the original matrix can be recreated using set of equations rather than matrices product. In order to reverse the process, the components being eliminated in the process of division and subtraction are to be collected and processed.

- On first operation of dividing the row by pivot, add the pivot to a stack A.
- On second operation of reducing the rows under the pivot element row, add to stack B the element multiplied to pivot before subtracting the corresponding row.

V. ALGORITHM TO CALCULATE INVERSE OF ROW REDUCTION

1. Calculate the echelon form of the matrix and collect along the information in stack A, B as specified in previous section.
2. Collect parameters from the original matrix and create a symbolic matrix of the same number of rows and columns.
3. Perform the same customized echelon algorithm on the symbolic matrix and collect reduced matrix and manipulated information in stack A and B.

4. Remove all the zeros of the echelon reduced matrix (not mandatory but they will not be used).
5. Create equation by equating the elements from reduced matrix and stack elements with corresponding elements from reduced symbolic matrix.
6. Solve the equation created in step 5; it will regenerate the original matrix again.

As a trivial modification the last element of last row can be left in reduced matrix instead of taking it to stack A. This approach has been used in section below.

A. Detailed Example:

Taking simple matrix,

$$\begin{pmatrix} 22 & 21 & 29 \\ 2 & 5 & 17 \\ 23 & 4 & 9 \end{pmatrix}$$

1. On passing it through echelon algorithm,

$$\begin{pmatrix} 1 & \frac{21}{22} & \frac{29}{22} \\ 0 & 1 & \frac{79}{17} \\ 0 & 0 & \frac{1056}{17} \end{pmatrix}, \begin{pmatrix} 2 & 23 & -\frac{395}{22} \end{pmatrix}, \begin{pmatrix} 22 & \frac{34}{11} \end{pmatrix}$$

2. Creating symbolic matrix with dimensions similar to original matrix,

$$\begin{pmatrix} a1 & a2 & a3 \\ a4 & a5 & a6 \\ a7 & a8 & a9 \end{pmatrix}$$

3. Applying customized row reduction on symbolic matrix,

$$\begin{pmatrix} 1 & \frac{a2}{a1} & \frac{a3}{a1} \\ 0 & 1 & \frac{a6-\frac{a3a4}{a1}}{a5-\frac{a2a4}{a1}} \\ 0 & 0 & a9-\frac{\left(a6-\frac{a3a4}{a1}\right)\left(a8-\frac{a2a7}{a1}\right)}{a5-\frac{a2a4}{a1}}-\frac{a3a7}{a1} \end{pmatrix},$$

$$\begin{pmatrix} a4 & a7 & a8-\frac{a2a7}{a1} \end{pmatrix}, \begin{pmatrix} a1 & a5-\frac{a2a4}{a1} \end{pmatrix}$$

Enumerating both matrices side by side generates,

$$\left(a4 = 2, \quad a7 = 23, \quad a8-\frac{a2a7}{a1} = -\frac{395}{22}, \quad a1 = 22\right)$$

$$\left(a5-\frac{a2a4}{a1} = \frac{34}{11}, \quad \frac{a6-\frac{a3a4}{a1}}{a5-\frac{a2a4}{a1}} = \frac{79}{17}, \quad \frac{a3}{a1} = \frac{29}{22}\right)$$

$$\left(a9 - \frac{\left(a6 - \frac{a3a4}{a1}\right)\left(a8 - \frac{a2a7}{a1}\right)}{a5 - \frac{a2a4}{a1}} - \frac{a3a7}{a1} = \frac{1056}{17}, \quad \frac{a2}{a1} = \frac{21}{22} \right)$$

Solving these equations together returns the original matrix,

$$\begin{pmatrix} a1 = 22 & a2 = 21 & a3 = 29 \\ a4 = 2 & a5 = 5 & a6 = 17 \\ a7 = 23 & a8 = 4 & a9 = 9 \end{pmatrix}$$

VI. EXPERIMENT

The experiment perform on the 4*4 matrix for example

$$\begin{pmatrix} 18 & 6 & 8 & -7 \\ 3 & -5 & 13 & 1 \\ 2 & 1 & -6 & 3 \\ 2 & -2 & 4 & -2 \end{pmatrix}$$

Consider this matrix above as original matrix. If this matrix is passed through WDR encoding and decoded at the other end, the approximate matrix obtained is:

$$\begin{pmatrix} 18 & 6 & 8 & -6 \\ 2 & -4 & 12 & 0 \\ 2 & 0 & -6 & 2 \\ 2 & -2 & 4 & -2 \end{pmatrix}$$

In the next section we discuss how we apply method on this original matrix and pass it through WDR algorithm in parts and decoding finally to achieve approximate original matrix again. Comparison is established between decoded matrix obtained from application of WDR on complete matrix at once and application on parts of matrix that we propose using three parameters PSNR, MSE, MAE, shown in table 1.

VII. APPLICATION OF CUSTOMIZED ROW REDUCTION ON WDR ALGORITHM

Since WDR converts the 2D array to 1D array, it is easy to first reduce the matrix using customized row reduction and then apply encoding one-by-one on all the decompositions obtained. This would help in avoiding maintaining all the values in RAM all the time. Customized row reduction is applicable on square matrix as well as rectangular matrix. For example,

$$\begin{pmatrix} 18 & 6 & 8 & -7 \\ 3 & -5 & 13 & 1 \\ 2 & 1 & -6 & 3 \\ 2 & -2 & 4 & -2 \end{pmatrix}$$

Applying the customized row reduction we get,

$$\begin{pmatrix} 1 & 0.33 & 0.44 & -0.87 \\ 0 & 1 & -1.94 & -0.36 \\ 0 & 0 & 1 & -0.62 \\ 0 & 0 & 0 & -3.48 \end{pmatrix},$$

$$\begin{pmatrix} 3 & 2 & 2 & 0.33 & -2.66 & -2.07 \end{pmatrix},$$

$$\begin{pmatrix} 18 & -6 & -6.24 \end{pmatrix}$$

First matrix can be converted into 1D array by collecting the values according to scan order and rest 1D arrays can also be rearranged based on their position in original matrix corresponding to scan order. WDR can be performed on each array separately. On decoding after encoding, the data received is,

$$\begin{pmatrix} 1 & 0.25 & 0.25 & -0.75 \\ 0 & 1 & -1.75 & -0.25 \\ 0 & 0 & 1 & -0.5 \\ 0 & 0 & 0 & -3.25 \end{pmatrix},$$

$$\begin{pmatrix} 3 & 2 & 2 & 0 & -2.5 & -2 \end{pmatrix},$$

$$\begin{pmatrix} 18 & -6 & -6 \end{pmatrix}$$

On using the reverse of Echelon method as purposed we get,

$$\begin{pmatrix} 18 & 4 & 4 & -14 \\ 3 & -5 & 11 & 4 \\ 2 & 0 & -6 & 4 \\ 2 & -2 & 3 & 0 \end{pmatrix}$$

Though this methods tried to solve the problem of encoding the matrix in parts but in case at the receiving end while decoding there is shortage of RAM again while solving equations, it is not necessary to load all equation at once in RAM, few of them can be taken and solved as far as possible, adding more equations subsequently and solving further. Our successful experiment on progressively solving equations revealed that to get the desired result, the equations must be arranged in increasing order in matrix starting from first row and element of matrix and only those unknowns should be considered that fall in those rows in symbolic matrix. The list of equations must be arranged in order for both customized row-reduced numerical matrix and customized row-reduced symbolic matrix in position as, [[1, 1], [1, 2], [1, 3], [1, 4], [2, 1], [2, 2], [2, 3], [2, 4], [3, 1], [3, 2], [3, 3], [3, 4], [4, 1], [4, 2], [4, 3], [4, 4]]. Now these groups of equations and their unknowns can be as small as 4 for each progressive step in this example.

VIII. RESULT AND CONCLUSION

The performance of this algorithm is evaluated using three parameter MSE (Mean Square Error), PSNR (Peak signal to noise ratio), Mean Absolute error.

MSE (Mean Square Error): Mean Square Error represents the average of the square of errors between original matrix and decompressed matrix. MSE should be minimum. Small MSE values mean output is of good quality [11].

$$MSE = \frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}(A_{i,j} - B_{i,j})^2 \qquad (1)$$

'A' represents the matrix data of our original data and 'B' represents the matrix data after decompressing the data. M and N are the number of rows and columns of an image. I and J is the index of rows and columns of matrix.

PSNR (Peak Signal to Noise Ratio): PSNR represents the peak error. PSNR should be high because it shows that signal-to-noise is high. PSNR is usually expressed in terms of the logarithmic decibel scale [11].

$$PSNR = 10log\frac{255^2}{MSE} \qquad (2)$$

Mean absolute Error: It is used to measure how close forecasts or our predictions are to the eventual outcomes. Mean Absolute error is given by [12].

$$MAE = \frac{1}{N}\sum_{i=1}^{N}|A_i - B_i| \qquad (3)$$

Table 1 Performance between WDR on Original Matrix and WDR on Row Reduced Matrix

Parameter	**WDR on Original Matrix**	**WDR on row reduced matrix**
PSNR	51.7210	40.6781
MSE	0.4375	5.56
MAE	1	7

Hence, it is possible to not pre-decide on how many partitions are to be made of the original matrix to attack the problem of decreasing RAM consumption in WDR algorithm processing. This technique is applicable on all types of matrices. Consumption of RAM is reduced by almost 50% thought number of repetition will increase by a factor of 3 to accommodate this advantage. The complexity of entire process gets increased by O (n^3) because of row reduction. As possible improvements to the program, one can pre-calculate the symbolic equations being mapped against the original row reduced matrix and map them against each other on the fly every time as needed. Another application of algorithm of customized row reduction is to generate extensive equations and check the authenticity of equation solving system by matching the equality between original matrix and decoded matrix. Because of the recent advances in equation solving [10] it is possible to solve these equations in relatively shorter period of time.

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