

International Journal of Computer Science and Mobile Computing



A Monthly Journal of Computer Science and Information Technology

ISSN 2320-088X

IJCSMC, Vol. 4, Issue. 7, July 2015, pg.438 – 445

RESEARCH ARTICLE

A HYBRID DIFFERENTIAL EVOLUTION METHOD FOR THE DESIGN OF HIGH PASS DIGITAL FIR FILTER

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Abstract— *This paper introduces a stochastic evolutionary algorithm known as hybrid Differential Evolution (DE) for the design of digital High pass FIR filter. It combines the features of both the basic DE and exploratory move for the fine tuning of DE parameters locally as well as globally in the promising search area. A multi-objective function having different performance requirements of minimum magnitude error and minimum pass band and stop band ripples is created to obtain the desired high pass digital FIR filter as close as possible to the ideal FIR filter. From the simulation results, it is confirmed that the proposed algorithm gives more accurate and better results.*

Keywords— *Digital Signal Processing (DSP), Finite Impulse Response (FIR) Filter, Differential Evolution (DE), exploratory move*

I. INTRODUCTION

Digital signal processing (DSP) is the mathematical manipulation of an information signal to modify or to improve it [1]. The objective of DSP is to measure and filter the analog signals. The first step is to convert the analog signal into digital form, then sampling and digitizing it by using an analog to digital converter (ADC). After applying signal processing techniques and algorithms using digital signal processor the digital signal is converted back to analog form by using digital to analog converter (DAC). The digital signal processor may be composed of resistors, capacitors or integrated circuits to accomplish a more difficult task. The applications of digital signal processing are more accurate, less sensitive to tolerances of component values, easily transported, independent of temperature, ageing and other external parameters. DSP is mainly used in audio and video signal processing, seismology, biomedicine and filter applications.

Filters are basically frequency selective devices which generally pass certain range of frequencies while rejecting others. Depending on the frequency range they pass, filters are categorized into four main categories: Low pass, High pass, Band pass and Band stop. Depending on the type of input signal, filters are of two types: analog and digital filters. Analog filters operate on continuous time signals and composed of imperfect electronic components such as resistors, inductors, capacitors and amplifiers etc. But the component values are affected by variations in environment conditions like temperature, humidity etc. So analog filters are very difficult to maintain. On the other hand, digital filters operate on digital signals and are composed of

components such as ADC, digital signal processor and DAC. Digital filters are usually software based, have better signal-to-noise ratio, definite filter coefficients and flexible [2].

There are two fundamental types of digital filters based on their impulse response: Finite impulse response (FIR) and Infinite impulse response filters (IIR) [3]. IIR filters are the filters whose impulse response extends over an infinite duration of time. The values of the output samples depend on present and past inputs and past outputs. So these are also known as recursive or feedback filters. But FIR filters have impulse response of only finite duration and the output is the function of only previous and present input samples, not of the past outputs. So they are known as non-recursive or feed forward filters. It is always possible to design an FIR filter transfer function with an exact linear-phase response, while it is nearly impossible to design a linear-phase IIR filter transfer function [4]. So this paper concentrates on the design of High pass digital FIR filter owing to its many advantages over IIR filters.

Various techniques to design digital FIR filters are: window method, frequency sampling and optimization. Window method is used to change infinite non causality sequence into finite impulse response sequence [5]. First, a proper ideal frequency selective response is chosen and then it is truncated to obtain the desired FIR filter. This method is most commonly used due to its simplicity and efficiency. The designing of filters using frequency sampling method is very easy but the designed filter has small attenuation in stop band and suffers from a lack of control over the frequency domain characteristics [5]. The frequency sampling methods give errors at the point where it is not sampled. So to overcome these drawbacks, Optimization technique is used for the remaining points.

Manira Khatun (2014) implemented new window function based on the Blackman window function for the design of FIR digital filter. This window function results in increased main lobe width by reducing transition width. But the ripples also increased [6]. Aggarwal *et. al.* (2012) implemented Hamming window method, Parks-McClellan method and Genetic Algorithm (GA) for the design of Low pass digital FIR filter. It has shown that filter design using GA is best because of its less transition bandwidth, small amount of ripples in pass band and stop band and better phase response as compared to other techniques. Most of the zeroes lie inside the unit circle due to which the phase delay is minimum [7]. Vijay. P *et. al.* (2012) implemented Particle Swarm Optimization (PSO) for low pass FIR filter design and compared it with various window methods. This method is superior to window method in terms of its fast convergence speed and minimum error. It is also better than GA in terms of complexity due to the absence of operators like mutation and crossover [8]. Kumar *et. al.* (2013) presented the hybrid of GA and PSO for low pass FIR filter which is better than individual GA and PSO in terms of efficiency as well as numerical stability. But if the initial parameters are not chosen correctly due to any reason, then it results in trapping in local minima [9]. So to overcome these drawbacks, Differential Evolution is used in this paper.

DE algorithm was firstly introduced by Storn and Price in 1995. It can be applied for nonlinear, non differentiable and non-convex functions. It provides the advantage of few control parameters and can find the global minimum independent of the initial parameter values. It uses the basic operators of evolution like mutation, crossover and selection as in GA. But the control parameters are less than that in GA which are: Population Size, Mutation Factor and Crossover Rate. Different mutation strategies are available for evolution in DE. This paper focuses on the design of High pass digital FIR filter using Hybrid DE which simultaneously explores and exploits the search space. The minimum magnitude error, pass band and stop band ripples are achieved by optimizing the filter coefficients.

The rest of the paper is structured into four sections. Section II illustrates the problem formulation for the High pass digital FIR filter design. Section III explains the detailed methodology for Hybrid DE which gives the mechanism of basic DE and exploratory move. The Section IV depicts the simulation results for the High Pass FIR filter. Finally, the conclusion is specified in Section V.

II. PROBLEM FORMULATION FOR HIGH PASS DIGITAL FIR FILTER

FIR filters are digital filters with finite impulse response. They are also known as non-recursive digital filters as they do not have the feedback i.e. a recursive part of a filter. For the phase characteristic of a FIR filter to be linear, the impulse response must be symmetric or anti-symmetric, which is expressed in the following way:

$$h(n) = h(N - n - 1); \quad \text{symmetric impulse response}$$

$$h(n) = -h(N - n - 1); \quad \text{anti-symmetric impulse response}$$

This paper deals with the symmetric property of even order FIR filter. So our design problem requires the values of $(N/2 + 1)$ number of coefficients in order to obtain the high pass digital FIR filter with certain characteristics.

The difference equation for digital FIR filter is as below:

$$y(n) = h(0)x(n) + h(1)x(n - 1) \dots \dots \dots + h(N)x(n - N) \tag{1}$$

The order of the filter is N , while the length of the filter (which is equal to the number of coefficients) is $N + 1$.

The transfer function of digital FIR filter is given as:

$$H(z) = \sum_{n=0}^N h(n) z^{-n} \tag{2}$$

Where $H(z)$ is the frequency domain representation of impulse response and $h(n)$ is the time domain representation of impulse response. N is the order of the filter.

For a digital High pass FIR filter the ideal response is defined as:

$$H_i(e^{jw}) = \begin{cases} 0 & \text{for } w \leq w_c \\ 1 & \text{otherwise} \end{cases} \tag{3}$$

The performance of High pass digital FIR filter is evaluated by using L_1 -norm and L_2 -norm approximation error of magnitude response and ripple magnitude of both pass-band and stop-band. The High pass digital FIR filter is designed by optimizing the filter coefficients.

For optimization, we have to minimize the L_p -norm approximation error function. L_p -norm is expressed as :

$$E(x) = \left\{ \sum_{i=0}^k |H_d(w_i) - |H(w_i, x)||^p \right\}^{1/p} \tag{4}$$

where $H_d(w_i)$ is the desired magnitude response of the ideal digital FIR filter and $H(w_i, x)$ is the obtained magnitude response of the FIR filter.

For $p=1$, magnitude error denotes the L_1 -norm error which is stated below:

$$e_1(x) = \sum_{i=0}^k |H_d(w_i) - |H(w_i, x)|| \tag{5}$$

For $p=2$ magnitude error denotes the L_2 -norm error which is given as:

$$e_2(x) = \left\{ \sum_{i=0}^k |H_d(w_i) - |H(w_i, x)||^2 \right\}^{1/2} \tag{6}$$

The desired magnitude response $H_d(w_i)$ of high pass digital FIR filter is given as :

$$H_d(w_i) = \begin{cases} 1 & \text{for } w_i \in \text{pass band} \\ 0 & \text{for } w_i \in \text{stop band} \end{cases} \tag{7}$$

The ripple magnitude of pass band and stop band is expressed as :

$$\delta_p = \max\{|H(w_i, x)|\} - \min\{|H(w_i, x)|\} \quad \text{for } w_i \in \text{pass band} \tag{8}$$

$$\delta_s = \max\{|H(w_i, x)|\} \quad \text{for } w_i \in \text{stop band} \tag{9}$$

Four objective functions for optimization are:

$$O_1(x) = \text{Minimize } e_1(x)$$

$$O_2(x) = \text{Minimize } e_2(x)$$

$$O_3(x) = \text{Minimize } \delta_p$$

$$O_4(x) = \text{Minimize } \delta_s$$

The multi objective function is converted into single objective function :

$$\text{Minimize } f(x) = w_1 O_1(x) + w_2 O_2(x) + w_3 O_3(x) + w_4 O_4(x) \tag{10}$$

where w_1, w_2, w_3, w_4 are the weighting functions.

III. HYBRID DIFFERENTIAL EVOLUTION

In this paper, Hybrid Differential Evolution (DE) algorithm has been implemented to enhance the ability of global search space by searching the local neighbourhood of global solution. In this algorithm DE and Hooke-Jeeves exploratory move work as a global and local search technique respectively. DE is population based heuristic algorithm mainly used for nonlinear and non-differentiable continuous space functions. This method converges faster and with more certainty than any other global optimization technique [10]. This algorithm can

be implemented very easily and it requires negligible parameter tuning, which has made the algorithm reasonably popular. DE algorithm includes four steps namely initialization, mutation, crossover and selection.

A. Parameter Selection

The optimization performance depends largely on the selection of control parameters such as population size (P), boundary constraints of population, mutation factor (f_m), crossover rate (CR) and maximum number of iterations (T_{max}). This algorithm always starts with the population size (P) which remains same throughout the entire execution process. Each population vector is D-dimensional where D is the number of filter coefficients.

The whole population is represented in the form of matrix x_{ij}^t .

B. Initialization

Initially, the values associated with the population vector are selected randomly according to uniform probability distribution within the range [0,1]. But the entire search space should be covered. Let x_{ij}^t denote the j^{th} element of the i^{th} member of population at current iteration. This value is chosen within a certain limit $[x_L, x_H]$ according to the following relation:

$$x_L \leq x_{ij}^t \leq x_U$$

where x_L and x_U are lower and upper limits of the random variable respectively.

Individuals are generated within the specified limits according to the following equation:

$$x_{ij}^t = x_j^{min} + rand() (x_j^{max} - x_j^{min}) \tag{11}$$

where $(j = 1, 2, \dots, D; i = 1, 2, \dots, P)$

where rand() is the values of random numbers lying in the range [0,1].

C. Mutation

Mutation expands the entire search space. It is the process of creating a new vector from the existing population by adding weighted difference of two or more vectors to any other vector. For the generation of resultant or mutant vector, different mutation strategies exist out of which five are listed below:

$$Z_{ij1}^t = P_{R1j}^t + f_m (x_{R2j}^t - x_{R3j}^t) \tag{12}$$

$$Z_{ij2}^t = x_{Bj}^t + f_m (x_{R1j}^t - x_{R2j}^t) \tag{13}$$

$$Z_{ij3}^t = x_{ij}^t + f_B (x_{Bj}^t - x_{ij}^t) + f_m (x_{R1j}^t - x_{R2j}^t) \tag{14}$$

$$Z_{ij4}^t = x_{Bj}^t + f_m (x_{R1j}^t + x_{R2j}^t - x_{R3j}^t - x_{R4j}^t) \tag{15}$$

$$Z_{ij5}^t = x_{R5j}^t + f_m (x_{R1j}^t + x_{R2j}^t - x_{R3j}^t - x_{R4j}^t) \tag{16}$$

where R_1, R_2, R_3, R_4, R_5 are randomly chosen indices from the population, which are different from index i , f_m is a mutation control parameter, f_B is a coefficient responsible for the level of recombination. Z_{ij}^t represents the mutant vector.

D. Recombination

Successful solutions from the previous generations are incorporated in the third step called cross-over or recombination. The process of recombination plays a very important role in order to enhance the diversity of the population. The components of target vector are mixed with donor vector to produce trial vector. The crossover probability CR controls the fraction of parameters values that are copied from the mutant vector.

$$U_{ij}^{t+1} = \begin{cases} Z_{ij}^t & \text{if } (R_4(j) \leq CR) \text{ or } (j = R_5(i)) \\ P_{ij}^t & \text{if } (R_4(j) > CR) \text{ or } (j \neq R_5(i)) \end{cases} \tag{17}$$

Where $(i = 1, 2, \dots, P; j = 1, 2, \dots, D)$

and $U_{ij}^{t+1} = [U_{i1}^{t+1}, U_{i2}^{t+1}, \dots, U_{iN}^{t+1}]^T$ indicates the trial vector. $R_4(j)$ is the j^{th} evaluation of a uniform random number generation with [0,1]. CR is the crossover rate within [0,1].

E. Selection

This is the final step of the DE algorithm. To decide whether or not the trial vector will be a member of the next generation, it is compared with the target vector. If the trial vector yields smaller objective function than the target vector, then it will be a part of next generation. Otherwise, the existing target vector will be preserved. The algorithm will continue until any stopping criterion is arrived.

$$x_{ij}^{t+1} = \begin{cases} U_{ij}^{t+1} & (j = 1, 2, \dots, D) ; \text{if } (A(U_i^{t+1}) < A(X_i^t)) \\ X_{ij}^t & (j = 1, 2, \dots, D) ; \text{otherwise} \end{cases} \tag{18}$$

where A_j represents the fitness function.

F. Exploratory move

Hooke and Jeeves performed two types of moves: Exploratory move and pattern search method. The exploratory move obtains the information about any function in the neighbourhood of the current base point. The search directions are created iteratively in such a way that they completely span the search space [10]. Then the best point is perturbed in the positive and negative directions along the created search directions. The exploratory move is a success when the small value of objective function is obtained in any of the perturbed directions and the current base point is updated along it. But it is a failure if the value of the objective function is not improved and the current base point remains unchanged. The current point indicating filter coefficients is perturbed as follow:

$$X_i^n = x_i^o \pm \Delta_i u_i^j \quad ; (i = 1,2, \dots P; j = 1,2, \dots D) \tag{19}$$

$$\text{where } u_i^j = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases} \tag{20}$$

D represents number of variables.

The value of the objective function $A(x_i^n)$ is calculated as below:

$$X_i^n = \begin{cases} x_i^o + \Delta_i u_i & ; A(x_i^o + \Delta_i u_i) < A(x_i^o) \\ x_i^o - \Delta_i u_i & ; A(x_i^o - \Delta_i u_i) < A(x_i^o) \\ x_i^o & ; \text{otherwise} \end{cases} \tag{21}$$

When all the filter coefficients are explored and minimum value of the objective function is found, the algorithm stops.

IV. RESULTS AND DISCUSSION

The multi-objective design of High pass digital FIR filter using the Hybrid Differential Evolution optimization technique is considered in this paper. This section represents the work performed for the design of FIR high pass filter.

The high pass digital FIR filter is designed by setting 200 equally spaced points within frequency domain $[0, \pi]$. FIR filter is to be designed with the initial conditions described in Table I.

Table I: Design conditions for High Pass digital FIR filter

FILTER TYPE	PASS BAND	STOP BAND	MAX VALUE OF $H(\omega, x)$
High Pass	$0.8\pi \leq w \leq \pi$	$0 \leq w \leq 0.7\pi$	1

The high pass FIR digital filter is designed by using hybrid differential evolution algorithm with different mutation strategies. After that various parameters like order of filter, population size, mutation factor and crossover rate are varied. DE parameters used for design of FIR high pass filter are given in Table II.

Table II: DE algorithm Parameters

PARAMETER	PARAMETER VARIATION RANGE	OPTIMIZED PARAMETERS
Filter Order	20-50	42
Population Size	60-140	120
Mutation Factor	0.5-1.0	0.8
Crossover Rate	0.1-0.5	0.30

In this first five mutation strategies are applied for different even filter orders. The best mutation strategy with minimum objective function is found at strategy 4. For the selection of filter order, mutation strategy 4 is applied on filter order varying from 20 to 50. The maximum number of iterations are taken as 100 for this. The obtained objective function at different filter orders are given in Table III. From all filter orders, the order 42

yields minimum objective function which also fulfills the objective of this paper. So the design of high pass digital FIR filter is performed at filter order 42 which results in only half number of filter coefficients i.e. 22 due to symmetry property. After that, population size is varied from value 60 to 140 for mutation strategy 4 and filter order 42. The best population is obtained at 120. This population is selected to design digital high pass FIR filter having objective function value of 0.904886. The mutation factor is varied from 0.5 to 1.0 for population size 120, mutation strategy 4 at order 42. The minimum value of objective function is found at 0.8 mutation factor. The crossover rate value is varied from 0.10 to 0.50 at 0.8 mutation factor. The minimum objective function is obtained at 0.30 which is selected to design High pass FIR digital filter.

The variation in magnitude with respect to Normalized frequency has been drawn in Fig. 1 for order 42 of High Pass FIR filter. The Fig. 2 shows the graph between Magnitude Response in dB and normalized frequency to design High pass digital FIR filter and Fig. 3 shows the graph between Phase Response and normalized frequency of order 42 to design High pass FIR filter. Obtained filter coefficients are given in Table IV. Table V depicts the Statistical data for order 42 which shows minimum value, maximum value, average value and standard deviation of the objective function.

Table III: Design results of High Pass digital FIR Filter for different orders

FILTER ORDER	OBJECTIVE FUNCTION	MAGNITUDE ERROR 1	MAGNITUDE ERROR 2	PASS BAND PERFORMANCE	STOP BAND PERFORMANCE
20	5.521108	2.696023	0.332754	0.124862	0.122940
22	4.429965	2.018156	0.255221	0.130265	0.085059
24	4.17912	1.771156	0.257839	0.144744	0.070161
26	3.770991	1.939641	0.248509	0.070628	0.087777
28	2.669963	1.407046	0.174058	0.057427	0.051612
30	2.141908	1.091149	0.130986	0.059317	0.032772
32	2.010877	0.954949	0.131158	0.070012	0.024545
34	1.829092	0.999735	0.120865	0.036184	0.036515
36	1.301121	0.684063	0.080526	0.030235	0.024543
38	1.030493	0.547658	0.080784	0.019915	0.030008
40	1.097044	0.725793	0.082475	0.045157	0.007965
42	0.900548	0.497198	0.056513	0.024611	0.013945
44	1.058983	0.797489	0.095865	0.067059	0.008691
46	3.273429	2.719956	0.317429	0.105452	0.104936
48	1.551894	1.317378	0.189275	0.035817	0.073481
50	2.95727	1.623509	0.223499	0.130968	0.055162

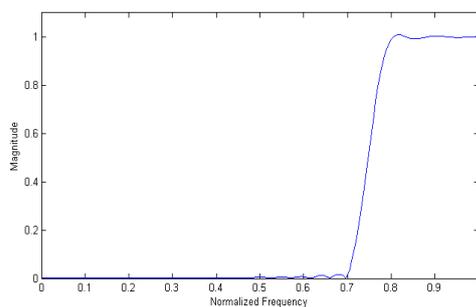


Fig. 1: Plot of Absolute Magnitude Response vs. Normalized Frequency

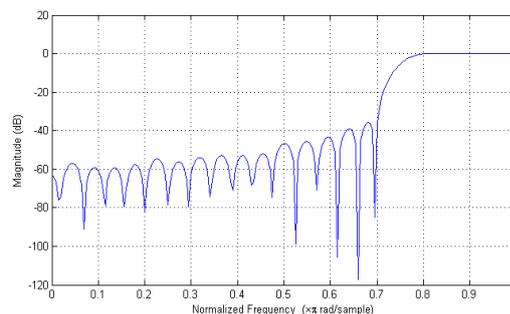


Fig. 2: Plot of Magnitude Response in dB and Normalized Frequency

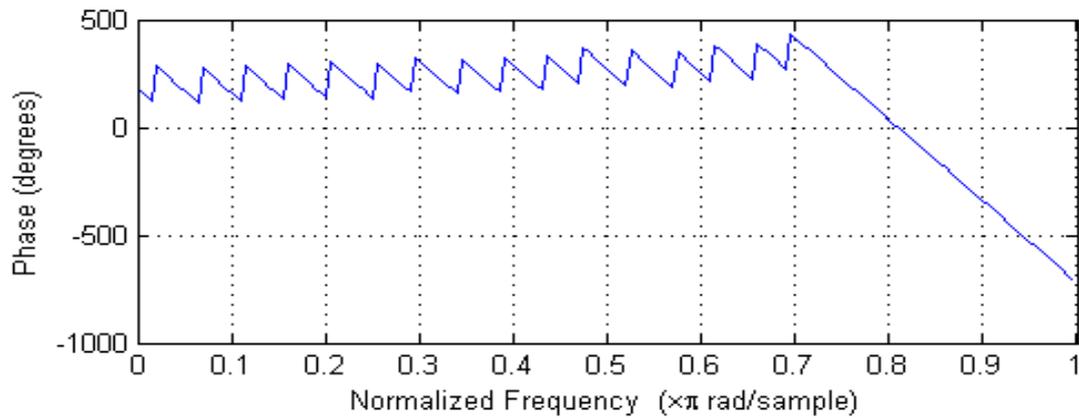


Fig. 3: Plot of Phase Response in dB vs. Normalized Frequency

Table IV: Filter coefficients obtained for designing High pass digital FIR filter.

h(n)	Filter Coefficients
h(0) = h(42)	-0.00216
h(1) = h(41)	0.001637
h(2) = h(40)	0.001448
h(3) = h(39)	-0.00532
h(4) = h(38)	0.006137
h(5) = h(37)	-0.00204
h(6) = h(36)	-0.00553
h(7) = h(35)	0.012059
h(8) = h(34)	-0.01186
h(9) = h(33)	0.002615
h(10) = h(32)	0.012142
h(11) =h(31)	-0.02317
h(12) = h(30)	0.021449
h(13) = h(29)	-0.00305
h(14) = h(28)	-0.02541
h(15) =h(27)	0.04737
h(16) = h(26)	-0.04386

$h(17) = h(25)$	0.003155
$h(18) = h(24)$	0.070603
$h(19) = h(23)$	-0.15717
$h(20) = h(22)$	0.226645
$h(21)$	-0.25324

Table V: Maximum, Minimum, Average value of objective function and Standard Deviation

Filter Type	Maximum Value	Minimum Value	Average Value	Standard deviation
High Pass	0.960612	0.908638	0.930982	0.011267

From the above results it is observed that the designed high pass digital FIR filter yields minimum value of L_1 – norm and L_2 – norm magnitude error, pass-band and stop band ripples.

V. CONCLUSIONS

The Hybrid DE is used to design the High pass digital FIR Filter by varying the various control parameters. The first five mutation strategies have been applied for different filter orders. On the basis of above results it can be concluded that it gives satisfied results for the filter order 42 at mutation strategy 4. Simulation results show better performance of the proposed algorithm in terms of magnitude error, maximum pass band and stop band performance. It is concluded that the maximum value, minimum value, average values of objective functions are 0.960612, 0.908638 and 0.930982 respectively. The achieved value of standard deviation is less than 1 which authenticates the robustness of the designed filter. Thus proposed algorithm is a useful tool for design of high pass FIR digital filter. The hybrid DE technique further can be extended to design low pass, band pass and band stop filters.

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