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RESEARCH ARTICLE

Stability Analysis and State Feedback Stabilization of Inverted Pendulum

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ABSTRACT - An inverted pendulum fitted on the moving cart (IPMC) represents a class of nonlinear system and is inherently unstable. Stabilizing such nonlinear systems is still a challenging objective for control engineers. It is very difficult to keep the pendulum always in upright position as it is highly sensitive to even a small disturbance and tend to fall down. Hence, the objective of the control mechanism is to maintain the pendulum up right position always by moving the cart back and forth, as long as the angle of deviation from upright position is within the admissible range. Several strategies such as Model Reference Control, Adaptive control, Sliding mode control, state feedback control etc., have already been proposed with reasonable accuracy for the control of IPMC problem. In the first part the mathematical model has been derived using fundamental principles, the controller has been designed using state feedback control technique and the entire system has been simulated using MATLAB-Simulink. The simulated results show that the performance of IPMC is satisfactory with state feedback control.

Keywords- Inverted pendulum, state feedback, nonlinear systems

1. INTRODUCTION

The inverted pendulum is a simple classic nonlinear mechanical device which models many physical systems. For example in Balancing a broomstick on our index finger or palm the position of our hand is constantly adjusted in order to keep the object at upright position . An inverted pendulum does basically the same thing. But it is limited in that it only moves in one dimension. The force must be applied to keep the system intact. The real life examples of inverted pendulum are Rockets and Missiles, Heavy Cranes lifting containers in shipyards, Self balancing Robots, and Future Transport Systems like Segways, Jetpack etc.... Some of the applications of 1. Inverted pendulum system are Simulation of dynamics of a robotic ARM, 2.Model of a human standing still. In the first example the inverted pendulum problem resembles the control systems that exist in robotic arms. The dynamic of Inverted Pendulum simulates the dynamics of robotic arm in the condition when the center of pressure lies below the center of gravity for the arm so that system is also unstable. Robotic arm behaves very much like inverted pendulum under this condition. In the second example a standing human looks like an inverted pendulum with the gravity center well above the ground. The feet of a standing human have plate like soles, rather than simple pivots. These plate-like soles are essential to stabilizing a standing posture. As an extreme counter example is the cruel three thousand years practice of foot wrapping of females in china until beginning of the 20^{th} century that made a woman tooter while walking.

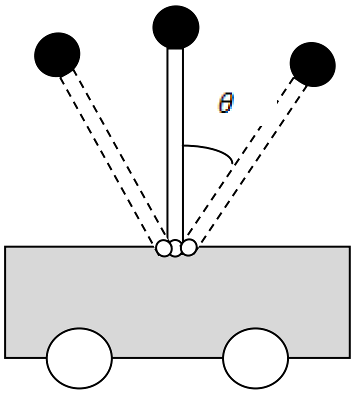


Fig 1: Schematic of an inverted pendulum on a moving cart

The IPMC is made up of a cart and a pendulum as shown in Fig (1). The goal of the controller is to move the cart to its commanded position without causing the pendulum to tip over. In open loop this system is unstable. The cart is driven by a DC motor or any other driving mechanism, and a pendulum is attached to the cart. The cart can move along a horizontal track, and the pendulum is able to rotate freely with respect to vertical in the vertical plane parallel to the track. By moving the cart front and back the pendulum can be kept at upright position. In this work the permanent magnet D.C motor is used for the cart movement. In the Figure.1.2 the mass of the cart and the pendulum are represented as M, m respectively. Several methods have been proposed to control the IPMC, such as traditional PID control, fuzzy control, genetic algorithm optimizing control, linear quadratic regulator etc. For higher order systems the PID control method is not suitable and it cannot directly be used in several more complicated cases, especially in MIMO (Multi Input Multi Output) systems. As the PID controller can control only one variable and the inverted pendulum system is one input-two output system and requires double PID controllers, the control strategy is complex. This article presents a state feedback control technique for stabilization of the inverted pendulum on a moving cart.

2. SYSTEM DESCRIPTION AND MODELLING

2.1 System Description

The IPMC has simple architecture as shown in the Fig 2. The physical system consists of cart, driven by a DC motor, and a pendulum attached to the cart. The cart can move along a horizontal track, and the pendulum is able to rotate freely with respect to vertical in the vertical plane parallel to the track. There is no direct control applied to the pendulum. In the Fig (2) the mass of the cart and the pendulum are represented as ‘M’ and ‘m’ respectively.

The objective is to maintain the pendulum at upright position, i.e. $\theta = 0$ with respect to perpendicular axis, by moving the cart forth and back. Since the equilibrium at θ=0 is unstable, such control objective has to be achieved while maintaining the stability of the system. The IPMC system inherently has two equilibrium points, one of which is stable while the other is unstable. The asymptotically stable equilibrium is when the pendulum is pointing downwards and in the absence of any control force the system naturally tends to return to this state. The unstable equilibrium however, is when the pendulum is standing exactly upwards and so a control force is needed to keep it in this position. The movement of the cart can be achieved by an ironless permanent magnet linear motor. Linear motor is a new type of driving device which can directly transform electric energy to mechanical linear motion. A control force exerted by the linear motor is required on the cart to maintain the pendulum in its upright equilibrium.

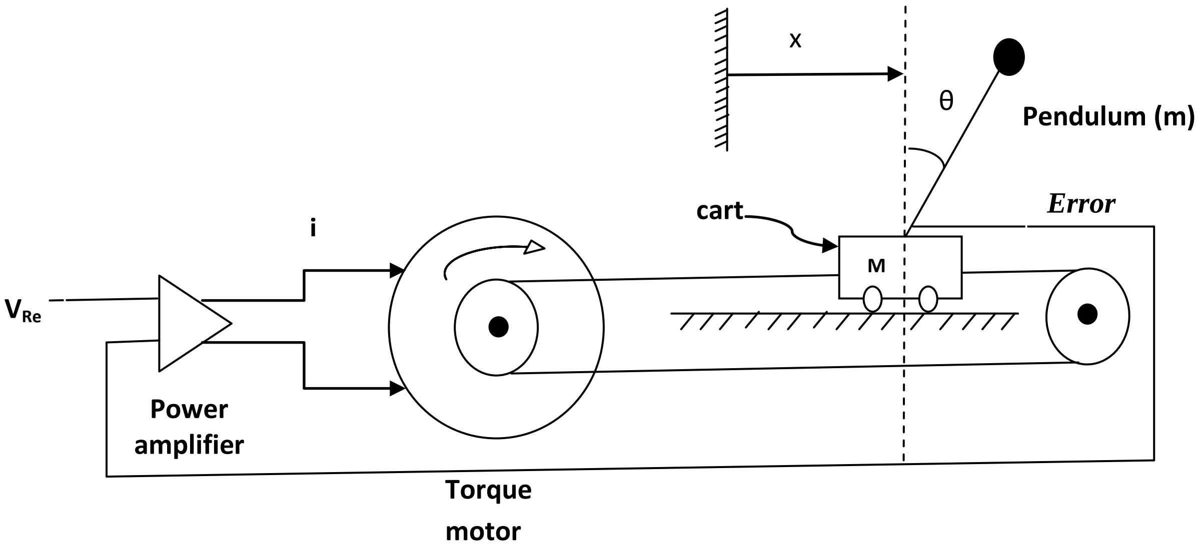


Fig 2: Schematic of an IPMC driven by the linear D.C motor

2.2 Nonlinear differential equations of system motion

A schematic of the inverted pendulum is shown in Fig. 3. In this paper, the simulation is based on a direct driven inverted pendulum system, in which a pendulum is mounted on a stage driven by an ironless permanent magnet linear motor.

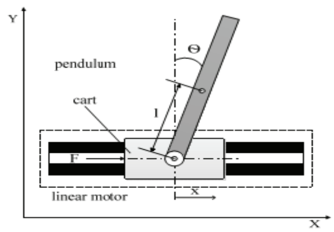


Fig 3: Schematic of IPMC driven by permanent magnet linear motor

The motion equations of the cart and pendulum in Fig (3) can be obtained using Hamilton's principle, shown in (1), (2):

$$(M+m)\ddot{x} - ml\sin\theta\,\dot{\theta}^2 + ml\cos\theta\,\ddot{\theta} = F_M + F_{fric} \quad (1)$$

$$\frac{4}{3}ml^2\ddot{\theta} + ml\ddot{x}\cos\theta = mgl\sin\theta \quad (2)$$

where x is the position of the cart, θ is the pendulum angle, measuring from the upright position, F is the force applied to the cart, $F_{fric}(t)$ is the friction force between cart and the track, and $f_{fric}(t)$ is the friction force between the pendulum and the pivot, which is very small thus ignored. In this section, since we concern the design of the inverted pendulum controllers, especially the dynamic performance of the controllers, the friction forces are simplified as viscous frictions, as in (3).

$$F_{fric} = -\epsilon\dot{x} \qquad (3)$$

In (3) ϵ is coefficient of viscous friction.

Table 1. Parameters in the inverted pendulum system

Parameter	Description	Value
M	Mass of the cart	1.336kg
m	Mass of the pendulum	0.083kg
L	Distance from the pivot to mass center of the pendulum	0.1685m
G	Gravitational constant	9.8m/s^2
K_f	Current to force conversion factor	$1 NperA$
ε	Coefficient of viscous friction	0.1N/m/sec

2.3 Mathematical model of system input force

The effective electromagnetic thrust applies by the linear motor to the cart is given by (4)

$$F_M = \frac{3\pi}{2\tau}[\lambda_{PM} i_q + (L_d - L_q) i_d i_q] \qquad (4)$$

where λ_{PM} is flux linkage generated by permanent magnet, L_d and L_q is flux linkage of d-axis and q-axis respectively. The linear motor adopts the zero d-axis current control method, thus the applied force can be simplified as (5).

$$F_M = K_f i_q \qquad (5)$$

In equation (5) i_q is the q-axis current after coordinate conversion from the external current supplied to the linear motor, and K_f is current to force conversion factor.

2.4 Mathematical model of inverted pendulum

Combine the equations (1), (2) and the input force equation (5), solve for $\ddot{x}$ and $\ddot{\theta}$ from the differential equations, and introduce the variable $z = (z_1 \quad z_2 \quad z_3 \quad z_4)^T = (x \quad \dot{x} \quad \theta \quad \dot{\theta})^T$ then after linearization the first-order mathematical model of the inverted pendulum system can be obtained as described in (6).

$$\dot{Z}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-4\varepsilon}{(4M+m)} & \frac{-3mg}{(4M+m)} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{3\varepsilon}{(4M+m)l} & \frac{3(M+m)g}{(4M+m)l} & 0 \end{bmatrix} Z(t) + \begin{bmatrix} 0 \\ \frac{4K_f}{4M+m} \\ 0 \\ \frac{-3K_f}{4M+ml} \end{bmatrix} i_q(t) \quad (6)$$

$$\dot{Z}(t) = AZ(t) + Bi_q(t)$$

3. APPLICATION OF STATE FEEDBACK TECHNIQUE TO IPMC

3.1. State feedback pole placement problem formulation:

The delay free completely controllable Linear time in variant system can be described as follows

$$\dot{\boldsymbol{X}} = \boldsymbol{A}\boldsymbol{X}(t) + \boldsymbol{B}u(t), X(t_0) = X_0 \quad (7)$$
$$y = \boldsymbol{C}\boldsymbol{X}(t)$$

Where $X \epsilon \mathcal{R}^n$ is the state vector and $u \epsilon \mathcal{R}$ is the control input.
$\boldsymbol{A}\ (n \times n)$ and $\boldsymbol{B}\ (1 \times n)$ are the system matrix and the control gain vector, respectively. $\boldsymbol{C}\ (1 \times n)$ is constant matrix. From A , the characteristic polynomial can be written as $\det(s\mathbf{I} - \boldsymbol{A}) = a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0 = 0$

where a=[a_0 a_1……a_n] , a_0=det(-**A**) and a_{n-1}=-trace(-**A**). The eigen values of the matrix $\det(s\mathbf{I} - \boldsymbol{A})$ determines the stability of the system. The control signal u can be chosen such that

$$u = -\boldsymbol{K}\mathbf{x}(t) \quad (9)$$

Where K is the designed feedback gain matrix. Therefore the control signal u is determined by instantaneous states. Such a scheme is called state feedback. The block diagram representation is shown in the below Fig (4). This closed loop system has no input. It objective is to maintain the zero input. Because of the disturbances that the output will deviate from zero. The nonzero output will be returned to the zero reference input because of the state feedback scheme of the system. Such a system where the reference input is always zero is called a regulator system.

3.2. Pole placement

If the system considered is completely state controllable, and if all the state variables are measurable then poles of the closed loop system can be placed at any desired locations by means of state feedback through an appropriate state feedback gain matrix. Let the desired closed loop poles are s=μ_1, s=μ_2, s=μ_3……. s=μ_n. By choosing appropriate gain matrix for state feedback, it is possible to force the system to have closed-loop poles at desired locations, provided that the original system is completely state controllable. From equation (7) the closed loop system can be represented as follows

$$\dot{\boldsymbol{X}} = (\boldsymbol{A} - \boldsymbol{B}\boldsymbol{K})\boldsymbol{X} \quad (10)$$

Then the solution of this equation is given by

$$\boldsymbol{X} = e^{(\boldsymbol{A}-\boldsymbol{B}\boldsymbol{K})}\boldsymbol{X}(0)$$

Where x(0) is the initial state caused by external disturbances. The stability and transient response characteristics are determined by the eigen values(also called as regulator poles) of matrix $\boldsymbol{A} - \boldsymbol{B}\boldsymbol{K}$. The problem of placing the regulator poles at The desired location is called as pole placement.

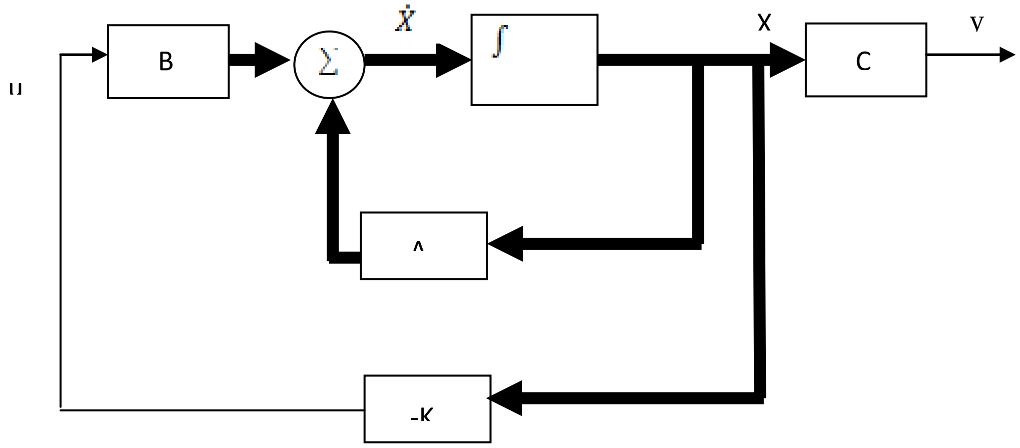


Fig 4: Schematic of regulator system

3.3. Determination of gain matrix k using ackermann's formula

The K can be determined by the following formula, $K = [0 \quad 0 \quad 1][B \quad AB \quad A^2B]^{-1}\phi(A)$ (11)

Where $\phi(A)$ is given by as follows

$$\phi(A) = A^n + \alpha_1 A^{n-1} + \cdots \alpha_{n-1}A + \alpha_n I = 0 \qquad (12)$$

3.4 Design of Type-1 Sservo system with plant has an integrator

Many control systems, require the system output to track an external reference input. In such cases, this necessitates modifying the design equation of the pole placement and the state observer. These types of systems are known as servo systems and as shown in the Fig (5). In this figure the integrator, together with state feedback scheme, is used to stabilize the system and asymptotically track step reference inputs with zero steady-state error. If the plant is defined by equation (7) with D=0, then the type 1 servo system can be designed by as follows.

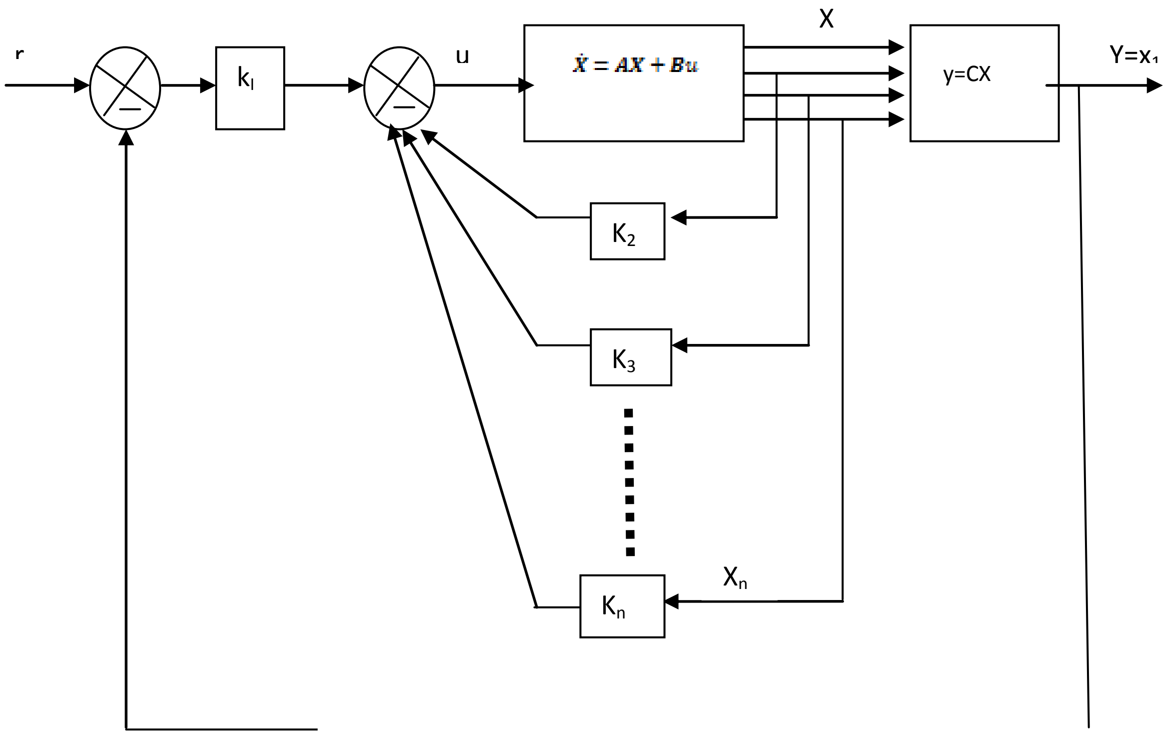


Fig 5: Block diagram of Type-1 servo system

The control signal u is given by

$$u = -\begin{bmatrix} 0 & k_2 & k_3 & \cdots & k_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + k_1(r - x_1) \qquad (13)$$

$$u = -\boldsymbol{KX} + k_1 r \qquad (14)$$

Where $\boldsymbol{K} = \begin{bmatrix} k_1 & k_2 & \dots & k_n \end{bmatrix}$ and r is the reference signal

Then equation (7) becomes

$$\dot{\boldsymbol{X}} = \boldsymbol{AX} + \boldsymbol{B}u = (\boldsymbol{A} - \boldsymbol{BK})\boldsymbol{X} + \boldsymbol{B}k_1 r \qquad (15)$$

And the error dynamics can be given by

$$\dot{\boldsymbol{e}} = (\boldsymbol{A} - \boldsymbol{BK})\boldsymbol{e} \qquad (16)$$

4. SIMULATION ANALYSIS OF IPMC

The developed mathematical model of state feedback (*Type 1* servo system) based IPMC is simulated using MATLAB-Simulink. The disturbance to the system is given as unit step. State signal z_1 represents the position of the cart and z_2 represents the velocity of the cart. State signal z_3 represents the angle of the pendulum and z_4 represents the angular speed. The output y, which is the deflection of pendulum with respect to vertical axis is assumed as one the state variables. Typical values assume for different components of the proposed inverted pendulum model is given in Table 1.

The state matrices **A**, **B** can be calculated using equation (6),

$$\dot{Z} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -0.0737 & -0.4339 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0.3284 & 45.550 & 0 \end{bmatrix} Z(t) + \begin{bmatrix} 0 \\ 0.73746 \\ 0 \\ -0.5599 \end{bmatrix} i_q(t)$$

And the output matrix is given by

$$y = \begin{bmatrix} x \\ \theta \end{bmatrix} = CZ = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix}$$

The matrix **C** can be calculate using the above equation

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} Z(t)$$

The settling time assumed is 3 seconds with a maximum peak overshoot of 15%.
Therefore, t_s= 3 sec and M_p=15%.

$$M_p = e^{-\pi\zeta/\sqrt{(1-\zeta^2)}} \qquad (17)$$

$$t_s = \frac{4}{\omega_n \zeta} \qquad (18)$$

From equations (17) and (18) ζ=0.5 , ω_n=2.6667 rad/sec. Desired closed loop poles are given by the following equation, $-\zeta\omega_n \pm j\omega_n\sqrt{(1-\zeta^2)}$ (22)

μ_1=-1.3+j2.25, μ_1=-1.3-j2.25, μ_1=-6.5, μ_1=-6.5.

The characteristics polynomial of the system matrix of the desired system is given by

$$\emptyset(s) = s^4 + 15.6s^3 + 82.807s^2 + 191.6s + 285.18$$

The gain matrix can be calculated using the Ackermann's formula and is given by

$$\boldsymbol{K} = [0 \quad 0 \quad 1]M^{-1}\emptyset(A)$$

Where $\emptyset(\boldsymbol{A})$ is obtained by substituting the system matrix in the characteristic equation of the desired system as follows

$$\emptyset(\boldsymbol{A}) = \boldsymbol{A}^4 + 15.6\boldsymbol{A}^3 + 82.807\boldsymbol{A}^2 + 191.6\boldsymbol{A} + 285.18\boldsymbol{I}$$

And M is controllability matrix and hence M^{-1} given by

$$[\boldsymbol{B} \quad \boldsymbol{AB} \quad \boldsymbol{A}^2\boldsymbol{B}]^{-1} = \begin{bmatrix} 0.0964 & 1.3659 & 0.000 & 0.0130 \\ 1.3659 & -0.0021 & 0.0120 & -0.0028 \\ -0.0022 & -0.0301 & -0.0171 & -0.0396 \\ -0.0300 & -0.0002 & -0.0396 & -0.003 \end{bmatrix}$$

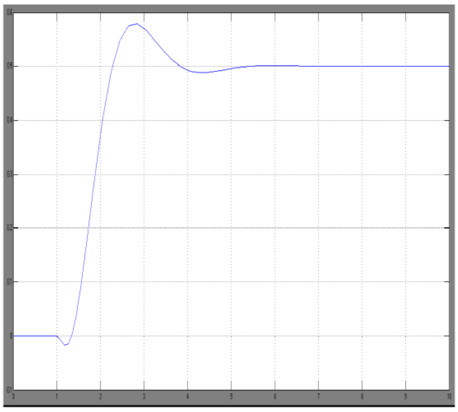
Therefore the feedback gain matrix is evaluated as

$$\boldsymbol{K} = [-8.55 \quad -7.5532 \quad -254.0307 \quad -37.6790]$$

$$k_1 = -8.55$$

5.1. Response of state signals:

cart position (m)



time (sec)

Fig 6: Response of the state signal z_1 -position of the cart

From Fig (6) to Fig (9) it is observed that the system is initially in stable state with pendulum in upright position. At time $t = 1$ the disturbance in the form of step is exerted at the reference input. This results in moving the cart from left to right i.e. in the negative direction as shown in the initial phase of Fig (6).

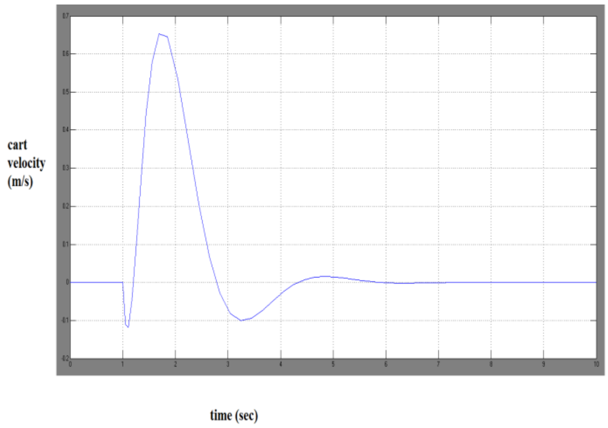


Fig 7: Response of the state signal z_2 - velocity of the cart

The velocity at which the cart moves and the angular velocity of the pendulum are given in Fig (7) and Fig (9) respectively and they follow the same pattern of response for obvious reasons. At time t=1 the speed of the cart is gradually increases up to 0.7 m/s and it is gradually decreases to zero in 3 sec.

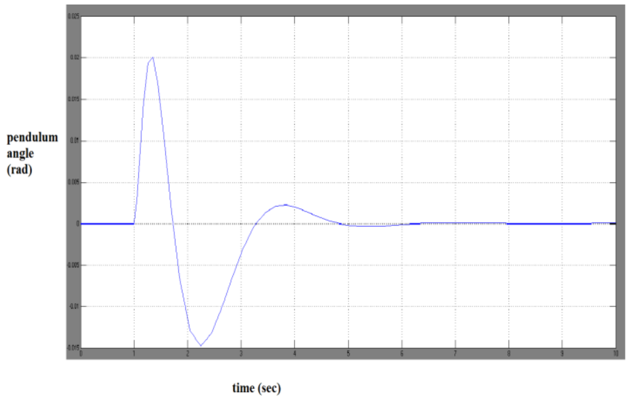


Fig 8: Response of the state signal z_3 – Angle (θ)

Fig (8) shows the plot of θ during the stabilization when the unit step disturbance is applied. The angle deviates from 0.02 to -0.02 radians in the time interval of 1 to 3 sec. The pendulum angle is stabilized in the 3 sec i.e., $\theta = 0^\circ$. Due to the movement of the cart from left to right it is obvious that the pendulum, because of its inertia, moves in the opposite direction to that of the cart movement and due its unstable nature the amount of deflection is relatively larger as shown in Fig (9). The error generated due to the deflection of the pendulum is fed to the state feedback controller and controller in turn generates the control signal to bring back the pendulum to upright position by applying force to the cart in the appropriate direction as shown in Fig (6) which makes the pendulum to move opposite direction and this process continues with forced required fall gradually. As per the design the settling time is found to be around 3 seconds as observed from Fig (9).

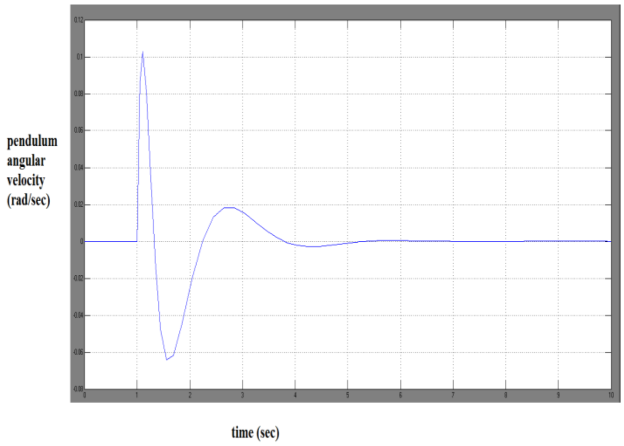


Fig 9: Response of the state signal z_3 – Angular velocity $\dot{\theta}$

The velocity of the angle deviates from 0.1 to -0.1 radians/sec in the time interval of $t = 1$ to 3 sec. And it is settled down to zero in 3 sec.

The pattern of control signal generated by the state feedback controller is given in Fig(10). It is noticed its magnitude is high when the error is more and gradually falls down to zero with oscillation in line with the magnitude and sign of error.

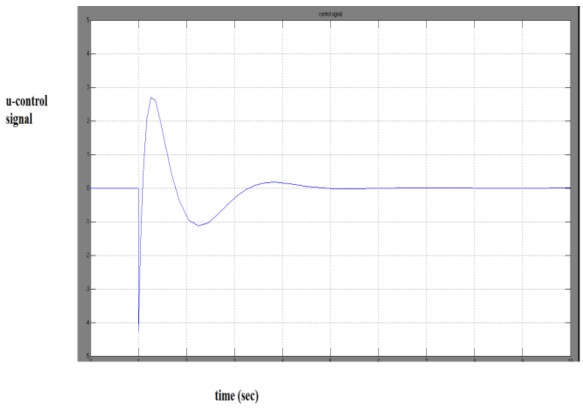


Fig 10: Response of control signal u

5. CONCLUSION AND FUTURE WORK

5.1. Conclusion

A complete analysis and design of state feedback controller to stabilize IPMC is presented in this work.. The IPMC problem is highly nonlinear and unstable and hence, the linear mathematical model of it is developed after linearizing the nonlinearities using Taylor series method and Type-I servo system based state feedback technique has been designed to stabilize IPMC. The designed controller is simulated in MATLAB-Simlink and results obtained show satisfactory performance as long as the disturbance is limited. To control cart movement a permanent magnet DC motor is proposed.

5.2. Future work

The state feedback methods proposed in this work is applied to the linearized model and without considering parameter uncertainties. A relatively new technique Quantitative Feedback Theory may be applied as it includes parameter uncertainties. IPMC is regulator problem and its stability is mainly influenced by parameter uncertainties that may arise due to model approximations and uncertainties and QFT addresses these issues effectively, QFT based controllers may yield better results than that of those offered by other controllers. However, estimating plant uncertainty and modeling environmental disturbances are challenging issues for a control engineer.

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