



RESEARCH ARTICLE

Critical Scan Path Based Energy Efficient LDPC Decoder using DD-BMP

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Abstract— A low-density parity-check (LDPC) code is a method of transmitting a message over a noisy transmission channel that are well known for their near-capacity error correction capabilities under iterative message-passing decoding. The inherently parallel nature of these codes allows very high throughput to be achieved. In the existing method energy-efficient architectures for decoders of low-density parity check (LDPC) codes using the Modified differential decoding with binary message passing (MDD-BMP) algorithm is used. This algorithm offer significant intrinsic advantages in the energy domain: simple computations, low interconnect complexity, and very high throughput. Using the MDD-BMP algorithm, these decoders achieve respective areas of 0.28mm², 1.38mm² and 15.37mm², average throughputs of 37 Gbps, 75 Gbps, and 141Gbps, and energy efficiencies of 4.9 pJ/bit, 13.2 pJ/bit, and 37.9pJ/bit with a 1.0 V supply voltage in post-layout simulations. At a reduced supply voltage of 0.8 V, these decoders achieve respective throughputs of 26 Gbps, 54 Gbps, and 94 Gbps, and energy efficiencies of 3.1 pJ/bit, 8.2 pJ/bit, and 23.5 pJ/bit. A new algorithm is proposed in which a scan path is added before input is processed which maximizes the throughput than existing method. For 1.0V the average throughputs are 39Gbps, 78Gbps and 145Gbps and for 0.8V the average throughputs are 29Gbps, 58Gbps and 97Gbps.

Index Terms— Binary messages, energy-efficient, high throughput, LDPC codes

1. INTRODUCTION

Low-density parity-check (LDPC) codes are a class of linear block LDPC codes. The name comes from the characteristic of their parity-check matrix which contains only a few 1's in comparison to the amount of 0's. Their main advantage is that they provide a performance which is very close to the capacity for a lot of different channels and linear time complex algorithms for decoding. Furthermore are they suited for implementations that make heavy use of parallelism. They were first introduced by Gallager in his PhD thesis in 1960. But due to the computational effort in implementing coder and decoder for such codes and the introduction of Reed-Solomon codes, they were mostly ignored until about ten years ago.

1.1 Representations for Ldpc Codes

Basically there are two different possibilities to represent LDPC codes. Like all linear block codes they can be described via matrices. The second possibility is a graphical representation. Matrix Representation. Let's look at an example for a low-density parity-check matrix first. The matrix defined in equation (1) is a parity check matrix with dimension $n \times m$ for a (8, 4) code. Now define two numbers describing these matrix. w_r for the number of 1's in each row and w_c for the columns. For a matrix to be called low-density the two conditions $w_c \ll n$ and $w_r \ll m$ must be satisfied. In order to do this, the parity check matrix should usually be very large, so the example matrix can't be really called low-density.

$$H = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix} \quad (1)$$

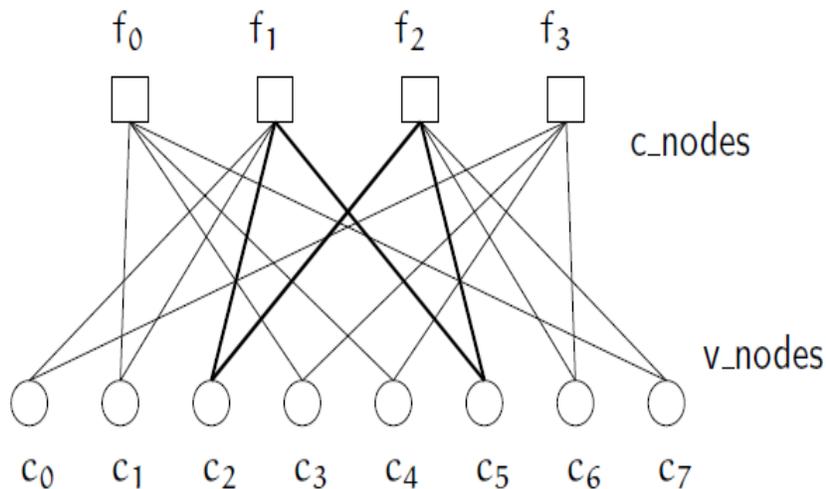


Figure.1 Tanner graph corresponding to the parity check matrix in equation (1). The marked path $c_2 \rightarrow f_1 \rightarrow c_5 \rightarrow f_2 \rightarrow c_2$ is an example for a short cycle. Those should usually be avoided since they are bad for decoding performance.

1.2 Regular and irregular LDPC codes

A LDPC code is called regular if w_c is constant for every column regular and $w_r = w_c \cdot (n/m)$ is also constant for every row. The example matrix from equation (1) is regular with $w_c = 2$ and $w_r = 4$. It's also possible to see the regularity of this code while looking at the graphical representation. There is the same number of incoming edges for every v-node and also for all the c-nodes.

If H is low density but the numbers of 1's in each row or column aren't constant the code is called an irregular LDPC code.

1.3 Constructing LDPC codes

Several different algorithms exist to construct suitable LDPC codes. Gallager himself introduced one. Furthermore MacKay proposed one to semi-randomly generate sparse parity check matrices. This is quite interesting since it indicates that constructing good performing LDPC codes is not a hard problem. In fact, completely randomly chosen codes are good with a high probability. The problem that will arise, is that the encoding complexity of such codes is usually rather high.

2. DECODING LDPC CODES

The algorithm used to decode LDPC codes was discovered independently several times and as a matter of fact comes under different names. The most common ones are the belief propagation algorithm, the message passing algorithm and the sum-product algorithm. In order to explain this algorithm, a very simple variant which works with hard decision, will be introduced first. Later on the algorithm will be extended to work with soft decision which generally leads to better decoding results. Only binary symmetric channels will be considered.

2.1 Hard-decision decoding

The algorithm will be explained on the basis of the example code already introduced in equation 1 and Fig.1. An error free received codeword would be e.g. $c = [1\ 0\ 0\ 1\ 0\ 1\ 0\ 1]$. Let's suppose that we have a BSC channel and the received the codeword with one error –bit c_1 flipped to 1.

1. In the first step all v-nodes c_i send a "message" to their (always 2 in our example) c-nodes f_j containing the bit they believe to be the correct one for them. At this stage the only information a v-node c_i has, is the corresponding received i -th bit of c , y_i . That means for example, that c_0 sends a message containing 1 to f_1 and f_3 , node c_1 sends messages containing y_1 (1) to f_0 and f_1 , and so on.

c-node	received/sent
f_0	received: $c_1 \rightarrow 1$ $c_3 \rightarrow 1$ $c_4 \rightarrow 0$ $c_7 \rightarrow 1$ sent: $0 \rightarrow c_1$ $0 \rightarrow c_3$ $1 \rightarrow c_4$ $0 \rightarrow c_7$
f_1	received: $c_0 \rightarrow 1$ $c_1 \rightarrow 1$ $c_2 \rightarrow 0$ $c_5 \rightarrow 1$ sent: $0 \rightarrow c_0$ $0 \rightarrow c_1$ $1 \rightarrow c_2$ $0 \rightarrow c_5$
f_2	received: $c_2 \rightarrow 0$ $c_5 \rightarrow 1$ $c_6 \rightarrow 0$ $c_7 \rightarrow 1$ sent: $0 \rightarrow c_2$ $1 \rightarrow c_5$ $0 \rightarrow c_6$ $1 \rightarrow c_7$
f_3	received: $c_0 \rightarrow 1$ $c_3 \rightarrow 1$ $c_4 \rightarrow 0$ $c_6 \rightarrow 0$ sent: $1 \rightarrow c_0$ $1 \rightarrow c_3$ $0 \rightarrow c_4$ $0 \rightarrow c_6$

Table 1: overview over messages received and sent by the c-nodes in step 2 of the message passing algorithm

2. In the second step every check nodes f_j calculate a response to every connected variable node. The response message contains the bit that f_j believes to be the correct one for this v-node c_i assuming that the other v-nodes connected to f_j are correct. In other words: If you look at the example, every c-node f_j is connected to 4 v-nodes. So a c-node f_j looks at the message received from three v-nodes and calculates the bit that the fourth v-node should have in order to fulfill the parity check equation. Table 2 gives an overview about this step. Important is, that this might also be the point at which the decoding algorithm terminates. This will be the case if all check equations are fulfilled. We will later see that the whole algorithm contains a loop, so an other possibility to stop would be a threshold for the amount of loops.

3. Next phase: the v-nodes receive the messages from the check nodes and use this additional information to decide if their originally received bit is OK. A simple way to do this is a majority vote. When coming back to our example that means, that each v-node has three sources of information concerning its bit. The original bit received and two suggestions from the check nodes. Table 3 illustrates this step. Now the v-nodes can send another message with their (hard) decision for the correct value to the check nodes.

v-node	y_i received	messages from check nodes		decision
c_0	1	$f_1 \rightarrow 0$	$f_3 \rightarrow 1$	1
c_1	1	$f_0 \rightarrow 0$	$f_1 \rightarrow 0$	0
c_2	0	$f_1 \rightarrow 1$	$f_2 \rightarrow 0$	0
c_3	1	$f_0 \rightarrow 0$	$f_3 \rightarrow 1$	1
c_4	0	$f_0 \rightarrow 1$	$f_3 \rightarrow 0$	0
c_5	1	$f_1 \rightarrow 0$	$f_2 \rightarrow 1$	1
c_6	0	$f_2 \rightarrow 0$	$f_3 \rightarrow 0$	0
c_7	1	$f_0 \rightarrow 1$	$f_2 \rightarrow 1$	1

Table 2: Step 3 of the described decoding algorithm. The v-nodes use the answer messages from the c-nodes to perform a majority vote on the bit value.

4. Go to step 2. loop

In our example, the second execution of step 2 would terminate the decoding process since c_1 has voted for 0 in the last step. This corrects the transmission error and all check equations are now satisfied.

2.2 Soft-decision decoding

The above description of hard-decision decoding was mainly for educational purpose to get an overview about the idea. Soft-decision decoding of LDPC codes, which is based on the concept of belief propagation, yields in a better decoding performance and is therefore belief the preferred method. The underlying idea is exactly the same as in propagation hard decision decoding. Before presenting the algorithm lets introduce some notations:

$$P_i = \Pr(c_i = 1|y_i) \tag{2}$$

- q_{ij} is a message sent by the variable node c_i to the check node f_j . Every message contains always the pair $q_{ij}(0)$ and $q_{ij}(1)$ which stands for the amount of belief that y_i is a "0" or a "1".
- r_{ji} is a message sent by the check node f_j to the variable node c_i . Again there is a $r_{ji}(0)$ and $r_{ji}(1)$ that indicates the (current) amount of believe in that y_i is a "0" or a "1".

The step numbers in the following description correspond to the hard decision case.

1. All variable nodes send their q_{ij} messages. Since no other information is available at this step, $q_{ij}(1) = P_i$ and $q_{ij}(0) = 1 - P_i$.

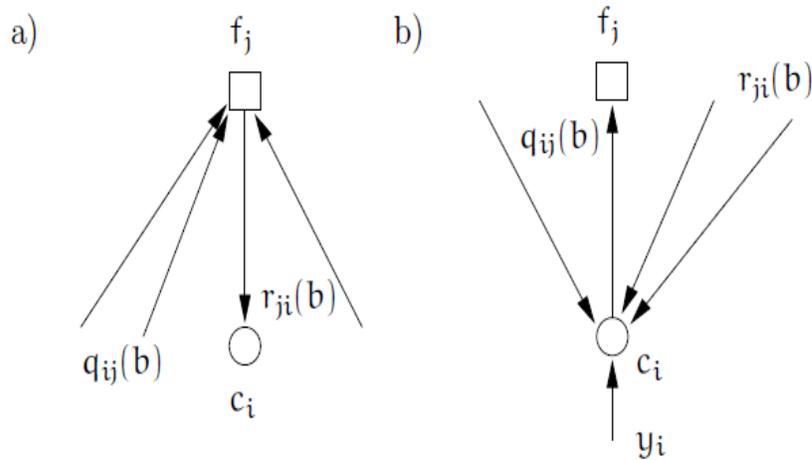


Figure 2: a) illustrates the calculation of $r_{ji}(b)$ and b) $q_{ij}(b)$

2. The check nodes calculate their response messages r_{ij}^2

$$r_{ji}(0) = \frac{1}{2} + \frac{1}{2} \prod_{i' \in V_j \setminus i} (1 - 2q_{i'j}(1)) \quad (3)$$

and

$$r_{ji}(1) = 1 - r_{ji}(0) \quad (4)$$

So they calculate the probability that there is an even number of 1's among the variable nodes except c_i (this is exactly what $V_j \setminus i$ means). This probability is equal to the probability $r_{ji}(0)$ that c_i is a 0. This step and the information used to calculate the responses is illustrated in figure 2.

3. The variable nodes update their response messages to the check nodes. This is done according to the following equations,

$$q_{ij}(0) = K_{ij} (1 - P_i) \prod_{j' \in C_i \setminus j} r_{j'i}(0) \quad (5)$$

$$q_{ij}(1) = K_{ij} P_i \prod_{j' \in C_i \setminus j} r_{j'i}(1) \quad (6)$$

whereby the Konstants K_{ij} are chosen in a way to ensure that $q_{ij}(0)+q_{ij}(1) = 1$. $C_i \setminus j$ now means all check nodes except f_j . Again figure 2 illustrates the calculation in this step. At this point the v-nodes also update their current estimation of their variable c_i . This is done by calculating the probabilities for 0 and 1 and voting for the bigger one. The used equations,

$$Q_i(0) = K_i (1 - P_i) \prod_{j \in C_i} r_{ji}(0) \quad (7)$$

and

$$Q_i(1) = K_i P_i \prod_{j \in C_i} r_{ji}(1) \quad (8)$$

are quite similar to the ones to compute $q_{ij}(b)$ but now the information from every c-node is used.

$$\hat{c}_i = \begin{cases} 1 & \text{if } Q_i(1) > Q_i(0), \\ 0 & \text{else} \end{cases} \quad (9)$$

If the current estimated codeword fulfills now the parity check equations the algorithm terminates. Otherwise termination is ensured through a maximum number of iterations.

4. Go to step 2. loop

The explained soft decision decoding algorithm is a very simple variant, suited for BSC channels and could be modified for performance improvements. Beside performance issues there are numerical stability problems due to the many multiplications of probabilities. The results will come very close to zero for large block lengths. To prevent this, it is possible to change into the log-domain and doing additions instead of multiplications. The result is a more stable algorithm that even has performance advantages since additions are less costly.

3. ALGORITHM (MDD-BMP)

Modified DD-BMP (MDD-BMP) is a variant of the original algorithm in which only a single memory is assigned to each variable node, replacing the unique variable-to-check messages with a single global message. Hence the memory update function is

$$M_v^{(k)} = M_v^{(k-1)} + \left[s \cdot \sum_{c' \in C_i} b_{c' \rightarrow v}^{(k-1)} \right]. \quad (10)$$

The variable node schematic for MDD-BMP is shown in Figure. 4. This simplification results in a loss in BER performance, though it also greatly reduces the complexity of the variable node. We demonstrate that in the case of MDD-BMP, the performance degradation is small, while the complexity reduction is significant, so using MDD-BMP in VLSI implementations is well justified. Furthermore, it allows the broadcasting concept to be applied to the variable-to-check messages.

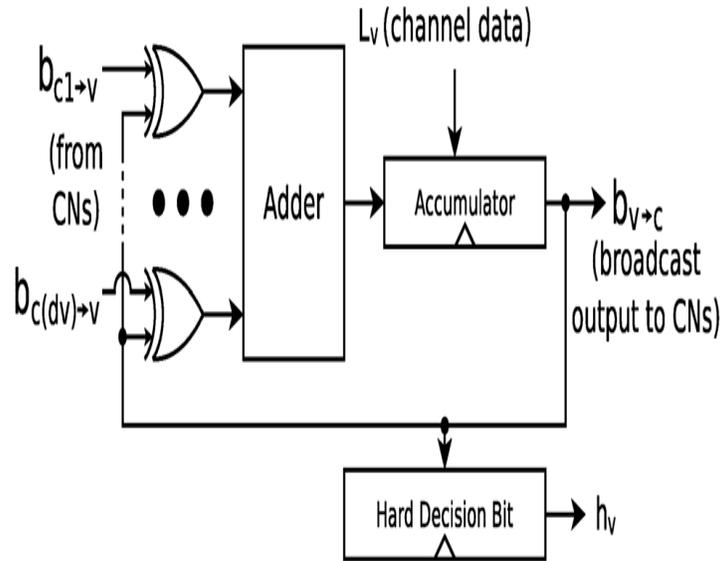


Figure.4. MDD-BMP variable node schematic.

In this work, we present results for decoders based on both DD-BMP and MDD-BMP. At the system level, these architectures possess several merits that are useful in energy-efficient designs. The variable and check nodes are both simple, which translates to lower area and power consumption in VLSI chips. Furthermore, binary message exchange and broadcasting reduces wiring overhead. Since several successive messages opposing the value stored in a VN must be received before the sign of the VN changes, switching activity is also low [12]. Furthermore, DD-BMP is amenable to efficient highly parallel architectures. Unlike some recent fully node-parallel decoder architectures such as bit-serial min-sum [5] and pulse width min-sum [7], iterations in DD-BMP complete in a single clock cycle. As a result, DD-BMP tends to converge to a valid codeword rapidly, giving it a very high average throughput. Voltage and frequency scaling (VFS) can be applied, trading off throughput for reduced dynamic energy consumption.

4. MDD-BMP WITH OTHER LDPC CODES

Although we have shown that differential binary (DB) algorithms are an effective, energy-efficient means of decoding FG-LDPC codes, it is not always practical or possible to use an FG code. In particular, communications standards mandate the use of standard codes, and there are currently no standards that use FG codes. Simulation results in [12] show that DD-BMP can effectively decode randomly generated LDPC codes, as long as the VN degree is high enough, but suffers from poor error correction performance otherwise. We have also verified that MDD-BMP has poor decoding performance for a (2048, 1723) RS-based LDPC code with (dv,dc) of (6,32) [17]. This code is used in the IEEE 802.3an standard for 10 Gbps Ethernet (10GBASE-T) [16], and is also a common benchmark code for highly parallel LDPC decoder implementations.

The related concept of *stopping sets*, which determine error floor performance of LDPC codes over the binary erasure channel, was studied in [21]. Error floor behaviour has also proven to be a major issue in the (2048,1723) RS-LDPC code, as the 10 Gbps Ethernet specification mandates very low BER performance. The dominant trapping sets of this code, termed *absorbing sets*, are defined as a special subclass of trapping set which is guaranteed to be stable under

Gallager bit-flipping decoding [22]. Additional analysis of the formation and dynamics of absorbing sets is performed in [23]. There have also been a number of implementation-oriented methods proposed for lowering the error floors of LDPC codes by overcoming or avoiding trapping sets. A prominent example is [10], which presents a hardware implementation of an OMS decoder for the (2048,1723) RS-LDPC code employing a post-processing decoding stage.

This technique has proven highly effective at overcoming the dominant (8,8) absorbing set of this code, and thereby lowering the error floor. Another hardware implementation in [24] proposes an iterative decoding algorithm with *backtracking*, which attempts to collapse trapping sets by identifying participating bits and flipping them. The stochastic decoders in [25] and [26] can employ *redecoding*, which restarts decoding with a different random number generator seed—since these algorithms are probabilistic, decoding may thus take a different trajectory and avoid trapping sets that caused a previous attempt to fail. Similarly, in *dithered belief propagation*, random processes are used in attempts to avoid or break out of trapping sets [27]. Likewise, early error floors and poor error correction performance are issues that must be solved for DB algorithms to be of practical use with general (non-FG) LDPC codes. In this work, we will focus on the (2048, 1723) RS-LDPC code, due to its importance to the IEEE 802.3an standard, the large body of prior work investigating its trapping sets, and its high popularity as an implementation target for highly-parallel decoder architectures.

Table 3

Wiring Complexity of The Implemented Decoders

Decoder	Total wire length (m)	Total interleaver wire length (m)	Interleaver wiring per edge (um)
DD-BMP	7920	3.586	772.69
MDD-BMP(273,191)	1793	0.337	72.53

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