



**RESEARCH ARTICLE**

# IMPLEMENTATION OF NORMALIZED CUT ALGORITHM FOR IMAGE SEGMENTATION

**Hardik K Patel<sup>1</sup>, Darshak G Thakore<sup>2</sup>, Mahasweta Joshi<sup>3</sup>**

<sup>1</sup>Research Scholar, Computer Engineering Department, BVM Engineering College, VV Nagar-388120, India

<sup>2</sup>Associate Professor, Computer Engineering Department, BVM Engineering College, VV Nagar-388120, India

<sup>3</sup>Asst Prof, Computer Engineering Department, BVM Engineering College, VV Nagar-388120, India

<sup>1</sup> [hardik404@gmail.com](mailto:hardik404@gmail.com); <sup>2</sup> [darshak\\_thakore@rediffmail.com](mailto:darshak_thakore@rediffmail.com); <sup>3</sup> [sweta.ce2013@gmail.com](mailto:sweta.ce2013@gmail.com)

---

**Abstract**— *Image Segmentation is an important image processing technique which is used to analyse colour, texture etc. Image Segmentation is used to separate an image into several “meaningful” parts. Normalized cut (Ncut) is based on graph cut technique to solve the image Segmentation problems. Rather than just focusing on local features and their consistencies, Ncut consider the global impression of an image. We have applied Ncut algorithm, on many images and successfully segmented the images into meaningful parts.*

**Key Terms:** - *Normalized cut (Ncut); Active contour model (snake); mean shift image segmentation*

---

## I. INTRODUCTION

Every image is a set of pixels, dividing an image into sub partitions on the basis of some similar characteristics like colour, intensity and texture is called image segmentation. The goal of segmentation is to change the representation of an image into something more meaningful and easier to analyse. Image segmentation is normally used to locate objects and boundaries like lines, curves, etc. in image. In image segmentation, image is divided into some regions and in those regions each pixel is similar with respect to some of the characteristic such as the colour, intensity, or texture.

Segmentation can be done by detecting edges or points or line in the image. When the points are detected in an image, and then on the basis of similarities between any two points make them into separate regions. And in the case of the line detection technique, all the lines are detected and the similarities in between those lines then on the basis of the dissimilarities between the lines or curves in the image, the image are divided into two regions. And in the case of edge detection, the edges are detected in the image and after finding the edges in the image. Through this get a better segmented image. Even it is the old technique to segment the image now days this segmentation technique is used to segment the image. [11]

There are several image segmentation techniques, developed to segment the image in a better way. Image segmentation algorithms are developed based on two basic properties of intensity values:

- Discontinuity based image segmentation
- Similarity based image segmentation

### **Discontinuity based image segmentation**

In discontinuity based image segmentation approach the partition is based on some changes in gray level intensity of the image.

- Detection of isolated points
- Detection of lines
- Edge detection

### **Detection of Isolated Points**

An isolated point may be viewed as a line whose length and width are equal to one pixel. Isolated points in an image are those points which have abruptly different gray values than those of its surrounding pixels. A mask is utilized for point detection and involves highlighting the gray value difference.

### **Detection of lines**

Line may be embedded inside a single uniformly homogeneous region. In the lines, segment the image on the basis of lines in the image.

### **Edge detection**

Pixels at which the intensity of an image changes abruptly are called edge pixels. Edges are set of connected edge pixels. An edge essentially demarcates between two distinctly different regions.

### **Similarity based image segmentation**

In similarity based approach segmentation is done based on grouping of pixels based on some features.

- Thresholding
- Region based image segmentation
- K-Means Clustering

### **Thresholding**

This method is based on threshold value to turn a gray-scale image into a binary image. The key of this method is to select the threshold value (or values when multiple-levels are selected). [1]

### **Region based image segmentation**

The region based segmentation is partitioning of an image into areas of connected pixels through the application of homogeneity similarity criteria among sets of pixels. Each of the pixels in a region is similar with respect to some characteristics or computed property such as colour, intensity and texture.

### **K-Means Clustering**

The k-means algorithm is an iterative technique that is used to partition the image into  $K$  clusters. Clustering based image segmentation is the process of assigning a label to every pixel in an image such that pixels with the same label share certain visual characteristics. [16]

There are three methods are efficient and very popular for Image segmentation.

- Active contour model (snake)
- Mean shift image segmentation
- Graph partition image segmentation

### **Active contour model (snake)**

The idea behind active contours or deformable models, for image segmentation is quite simple. The user specifies an initial guess for the contour, which is then moved by image driven forces to the boundaries of the desired objects. The internal forces, defined within the curve, are designed to keep the model smooth during the deformation process, while the external forces, which are computed from the underlying image data, are defined to move the model toward an object boundary or other desired features within the image. One way of describing this curve is by using an explicit parametric form. [7]

### **Mean shift image segmentation**

The idea behind mean shift is to treat the points in the  $n$ -dimensional feature space as an empirical probability density function where dense regions in the feature space correspond to the local maxima or modes of the underlying distribution. For each data point in the feature space, one performs a gradient ascent procedure on the local estimated density until convergence. The stationary points of this procedure represent the modes of the distribution. Furthermore, the data points associated (at least approximately) with the same stationary point are considered members of the same cluster. Mean shift is a popular low-level segmentation technique for images and videos. It has been used in numerous applications such as noise removal, object tracking, 3D reconstruction, image and video stylization and video editing. [13]

### **Graph partition image segmentation**

In these methods, the image is modelled as a weighted, undirected graph. Usually a pixel or a group of pixels are associated with nodes and edge weights define the (dis)similarity between the neighbourhood pixels. The graph (image) is then partitioned according to a criterion designed to model "good" clusters. Each partition of the nodes (pixels) output from these algorithms are considered an object segment in the image. Some popular algorithms of this category are normalized cuts [2], random walker [3], minimum cut [16] and isoperimetric partitioning [4].

## II. GROUPING AS A GRAPH PARTITION

A graph cut is the process of partitioning a directed or undirected graph into disjoint sets.

### 2.1 Graph partition related approach

The set of points are presented as a weighted undirected graph  $G=(V,E)$ . An edge is formed between every pair of nodes and the weight on each edge  $W(i,j)$  is a function of the similarity between nodes  $i$  and  $j$ . A graph  $G=(V,E)$  is partitioned into two disjoint sets  $A$  and  $B$ ,  $B=V-A$ , by removing the edges connecting two parts. The degree of dissimilarity between two sets can be computed as a total weight of removed edges.

A graph  $G=(V,E)$  can be partitioned into two disjoint sets, by simply removing edges connecting the two parts. The degree of dissimilarity between these two pieces can be computed as total weight of the edges that have been removed. In graph theoretic language, called the **cut**.

$$cut(A, B) = \sum_{u \in A, v \in B} w(u, v) \quad (1) [2]$$

The optimal bipartition of a graph is the one that minimizes this cut value. Although there are an exponential number of such partitions, finding the minimum cut of a graph is a well-studied problem and there exist efficient algorithms for solving it.

Wu and Leahy proposed [1] a clustering method based on this minimum cut criterion. In particular, they seek to partition a graph into  $k$ -sub graphs such that the maximum cut across the subgroups is minimized. This problem can be efficiently solved by recursively finding the minimum cuts that bisect the existing segments. This globally optimal criterion can be used to produce good segmentation on some of the images. [15]

The minimum cut criteria favours cutting small sets of isolated nodes in the graph.

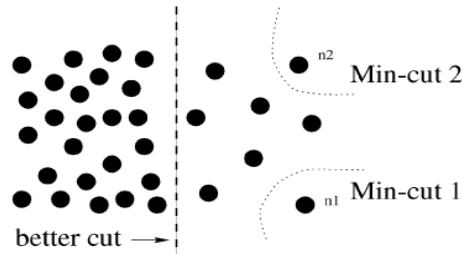


Figure 1 Case of minimum cut where isolated points are favoured [2]

To avoid this unnatural bias for partitioning out small sets of points, Shi and Malik [2] proposed a new measure of disassociation between two groups. Instead of looking at the value of total edge weight connecting the two partitions, they compute the cut cost as a fraction of the total edge connections to all the nodes in the graph and called this disassociation measure the normalized cut (Ncut).

### 2.2 Normalized cut (Ncut)

Shi and Malik [2] propose a modified cost function, normalized cut, to overcome the problem involved in minimum cut. Instead of looking at the value of total edge weight connecting the two partitions, the proposed measure computes the cut cost as a fraction of the total edge connections to all nodes. Mathematically it is;

$$Ncut(A, B) = \frac{cut(A, B)}{asso(A, V)} + \frac{cut(A, B)}{asso(B, V)} \quad 2 [2]$$

Where  $asso(A, V)$  is the total connection from nodes in  $A$  to all nodes in the graph and  $asso(B, V)$  is similarly defined.

$$asso(A, V) = \sum_{u \in A, t \in V} w(u, t) \quad 3 [2]$$

With this definition of the disassociation between the groups, the cut that partitions out small isolated points will no longer have small Ncut value, since the cut value will almost certainly be a large percentage of the total connection from that small set to all other nodes.

In similar fashion, a measure for total normalized association within groups for a given partition can be defined.

$$N_{assoc}(A, B) = \frac{assoc(A, A)}{assoc(A, V)} + \frac{assoc(B, B)}{assoc(B, V)} \tag{4[2]}$$

Where  $assoc(A,A)$  and  $assoc(B,B)$  are total weights of edges connecting nodes within A and B, respectively. This is also an unbiased measure, which reflects how tightly on average nodes within the group are connected to each other. Another important property of this definition of association and disassociation of a partition is that they are naturally related [1].

**2.3 Optimum computation of normalized cut**

Optimum computation for graph partitioning using the normalized cut as an optimal criterion can be found using eigenvector system. Given a partition of a graph  $V$  into two disjoint complementary sets  $A$  and  $B$ , let  $x$  be an  $N=|V|$  dimensional indication vector,  $x_i=1$  if node  $i$  is in  $A$ ,  $-1$  otherwise. Let  $d_i = \sum_j W(i,j)$  be the total connection from node  $i$  to all other nodes. Ncuts can be rewritten as .

$$Ncut(A, B) = \frac{cut(A, B)}{asso(A, V)} + \frac{cut(A, B)}{asso(B, V)} \tag{5 [2]}$$

$$= \frac{\sum_{x_i > 0, x_j < 0} -w_{ij} x_i x_j}{\sum_{x_i > 0} d_i} + \frac{\sum_{x_i < 0, x_j > 0} -w_{ij} x_i x_j}{\sum_{x_i < 0} d_i}$$

Let  $D = diag(d1, d2, \dots, dn)$  and  $W(i,j) = wij$  then Finding global optimum reduces to

$$\min_x Ncut(x) = \min_y \frac{y^T (D - W) y}{y^T D y}, \quad y_j \in \left\{ 1, \frac{-\sum_{x_i > 0} d_i}{\sum_{x_i < 0} d_i} \right\}, \quad y^T \begin{bmatrix} d_1 \\ \vdots \\ d_n \end{bmatrix} = 0 \tag{6 [2]}$$

If  $y$  is relaxed to take on real values, we can minimize above equation by solving generalized Eigen value system:

$$(D - W)y = \lambda Dy \tag{7 [2]}$$

Constraints on  $y$  come from the condition on the corresponding indicator vector  $x$ . The second smallest eigenvector of the generalized system satisfies the normality constraint.  $y$  is a real valued solution to this normalized cut problem . The constraint, that  $y$  takes on the two discrete values, is not satisfied.

**2.4 The Grouping Algorithm**

The eigenvector corresponding to the second smallest Eigen value is the real-valued solution that optimally sub-partitions the entire graph. The approximate Lanczos method is used for computation the eigenvector of a very sparse matrix where only couple of eigenvectors are needed. Computational cost is typically less than  $O(n1.5)$ , where  $n$  is the number of nodes in the graph.[17]

Graph is partitioned using the second smallest eigenvector, element value of which is continuous value. Normalized cuts are computed for each point. Splitting point is chosen with the point that gives the minimum Ncut value. The other approach could be to take 0, median or a mean value as a splitting point. Mean methods are good enough if values are very well separated. Recursively apply the algorithm to every sub graph until the Ncut exceeds certain limit. If a median value as a splitting point then it wasn't really reliable because of the approximation error. If a 0 value is taken as a splitting point then also error is generated.

Normalized cut algorithm Steps are summarized below [2]

1. Given a set of features, set up a weighted graph  $G = (V,E)$ , compute the weight on each edge, and summarize the information into  $W$ , and  $D$
2. Solve  $(D-W)x = \lambda Dx$  for eigenvectors with the smallest Eigen values.
3. Use the eigenvector with second smallest Eigen value to bipartition the graph by finding the splitting point such that Ncut is maximized
4. Decide if the current partition should be subdivided by checking the stability of the cut, and make sure Ncut is below pre-specified value

Recursively repartition the segmented parts if necessary.

### III. SUMMARIZING RESULTS

The different images are taken, applied the algorithm and generate following results.

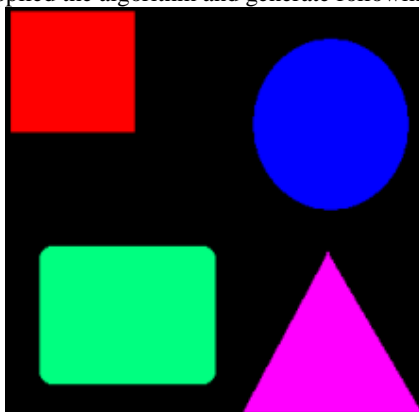


Figure 2: Input image



Figure 3: Grey-scale image of input image  
The output images are as follow.



Figure 4.1: Output images (1)

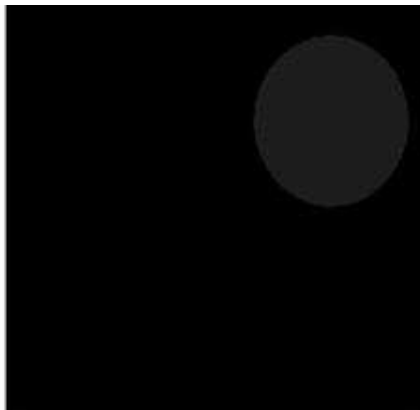


Figure 4.2: Output images (2)

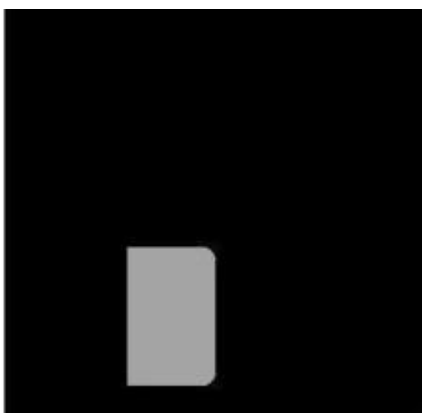


Figure 4.3: Output images (3)



Figure 4.4: Output images (4)

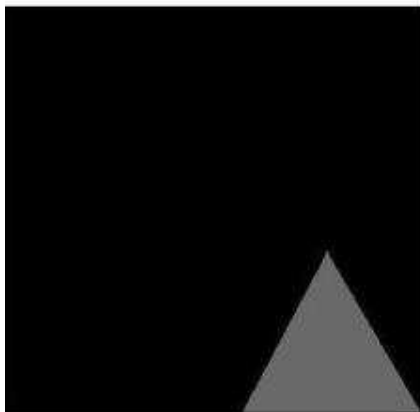


Figure 4.5: Output images (5)

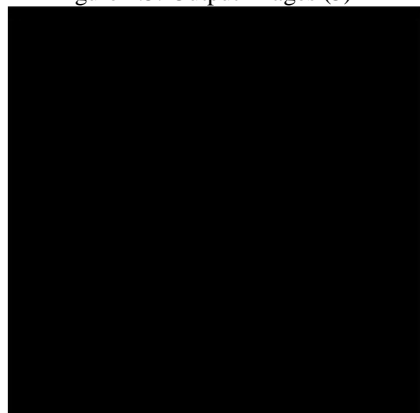


Figure 4.6: Output images (6)



Figure 5: Input image

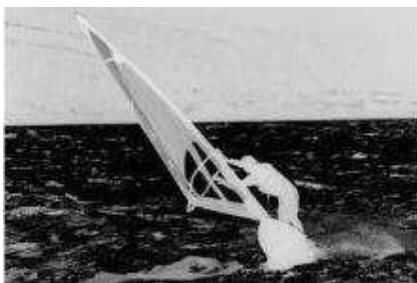


Figure 6: Grey-scale image of input image



Figure 7.1: Output images (1)

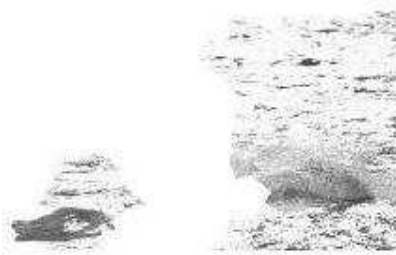


Figure 7.2: Output images (2)



Figure 7.3: Output images (3)



Figure 7.4: Output images (4)



#### IV. CONCLUSIONS

Ncut algorithm is accurately segment the given image into meaningful parts. Time complexity of algorithm is higher due to calculation of Eigen vector and Eigen values.

#### REFERENCES

- [1] J. Sijbers and K J. Batenburg, "Optimal Threshold Selection for Tomogram Segmentation by Projection Distance Minimization", IEEE Transactions on Medical Imaging, vol. 28, no. 5, pp. 676-686, June, 2009
- [2] Jianbo shi and jitendra malik (2000): "normalized cuts and image segmentation", IEEE transactions on pattern analysis and machine intelligence, pp 888-905, vol. 22, no. 8.
- [3] Leo grady and eric l. Schwartz (2006): "isoperimetric graph partitioning for image segmentation", IEEE transactions on pattern analysis and machine intelligence, pp. 469-475, vol. 28, no. 3.
- [4] Leo grady (2006): "random walks for image segmentation", IEEE transactions on pattern analysis and machine intelligence, pp. 1768-1783, vol. 28, no. 11
- [5] Linda G. Shapiro and George C. Stockman (2001): "Computer Vision", pp 279-325, New Jersey, Prentice-Hall, ISBN 0-13-030796-3
- [6] M. Fiedler, "a property of eigenvectors of nonnegative symmetric matrices and its application to graph theory", czech. Math. J. 25, pp.619--637, 1975.
- [7] Michael kass, andrew witkin and demetri terzopoulos," snakes: active contour models" international journal of computer vision, 321-331(1988)
- [8] Ohlander, ron; price, keith; reddy, d. Raj (1978). "picture segmentation using a recursive region splitting method". Computer graphics and image processing 8 (3): 313-333. Doi:10.1016/0146-664x(78)90060-6
- [9] Paresh tank seminar report on "image segmentation" BVM, dec2012
- [10] Pothen and h. Simon, "partitioning sparse matrices with eigenvectors of graphs", ibm journal of research and development, pp. 420-425, 1973.
- [11] S.vamsi, "image segmentation based on graph cut technique" thesis, department of computer science and engineering n i t rourkela orissa, india, may-2012
- [12] Shital Raut at al, "Image segmentation-State of art survey for prediction ", ICACC, IEEE -2008
- [13] Sylvain paris fr ´edo "a topological approach to hierarchical segmentation using mean shift" sparis, 2007.
- [14] Wu, z., and Leahy, r., "an optimal graph theoretic approach to data clustering: theory and its application to image segmentation", pam i (15), no. 11, pp. 1101-1113, 1993.
- [15] Yatharth saraf," algorithms for image segmentation" thesis birla institute of technology and science, pilani, may, 2006
- [16] Yu-Fei Ma, Hong-Jiang Zhang, Ming Luo "A Spatial Constrained K-Means Approach to Image Segmentation" mingluo-03
- [17] Baris Sumengen, Jelena Tešić "Normalized Cuts and Image Segmentation" ECE 278 Final Report