



# Study and Performance Analysis of IIR Filter for Noise Diminution in Digital Signal using MATLAB

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*Abstract: In area of Digital Signal Processing, the purpose of filters is to remove the selected range of signal such as noise or to extract meaningful data from the signal. A filter is a device which is designed to pass frequencies within a specific range while rejecting all other unwanted frequencies that fall outside this range. Filters are widely in use in the field of communication and signal processing applications such as radar, video operations, audio processing, ECG, EMG, EEG, channel equalization, signal filtering, noise reduction, analyzing of financial and economic data and image processing,. In the paper, Noise detected is quantization noise. Here, the word quantization is quite different from the terminology used during the conversion of signal from analog to digital. Here the word quantization is used for quantization of digital filter coefficients, and this quantization may either truncation or round-off type. The quantization noise is computed with help of MATLAB R-2012 software through Round off Noise Power Spectrum, and further this computed noise is reduced to its optimum values by using some methods. This paper present the study of IIR and FIR filter and evaluate the performance of different IIR filters such as Chebyshev-I, Chebyshev-II, Butterworth and elliptic filter with frequency 10 Hz to 30 KHz. MATLAB R-2012 software is used to simulate the different algorithm and result showed that Butterworth filter and Chebyshev-II filter gives improved result for 10Hz - 30 KHz frequency than other filter.*

**Keywords:-** IIR Filter, FIR Filter, Butterworth, Chebyshev-I, Chebyshev-II, Elliptic filter.

## INTRODUCTION

The basic functional need for filtering is to pass a range of frequencies while rejecting others. This need for filtering has many technical uses in the digital signal processing (DSP) areas of data communications, imaging, digital video, and voice communications. Analog filters are continuous-time systems for which both the input and output are continuous-time signals. Digital filters are discrete time systems whose input and output are discrete time signals. Digital filters are implemented using electronic digital circuits that perform the operations of delay, multiplication, and addition. Analog filters are implemented using resistors, inductors, capacitors, and, possibly, amplifiers. Digital filters can be implemented using integrated circuits so that per unit cost of digital filter construction is less than a comparable analog filter. Tolerances and accuracy considerations are important factors for both analog and digital signal processing. A system designer has much better control of accuracy of digital systems in terms of word length, floating point versus fixed-point arithmetic, and other similar factors. These are the major advantages of digital filters.

Infinite impulse response (IIR) digital filters are recursive (feedback type) systems that involve fewer design parameters, less memory requirements, and lower computational complexity than finite impulse response (FIR) digital filters.

### Filter Design Method

In this paper we will discuss about the filter design method and its characteristics. Four design methods are considered here i.e. Butterworth, Chebyshev-I, Chebyshev-II and Elliptic filters. Since, all filter type can be generated by its lowpass filter. Hence, only the lowpass filter of the aforesaid design methods are taken into account.

The general approach in obtaining a lowpass characteristic is to seek a function of the form are going make a attempt look back the various research work carried out in this field.

$$|H(j\omega)|^2 = \frac{A_0}{1 + F(\omega^2)} \quad (1)$$

Such that

$$\begin{aligned} F(\omega^2) &\ll 1 & 0 < \omega < \omega_p \\ F(\omega^2) &\gg 1 & \omega > \omega_s \end{aligned}$$

This, in turn, makes  $|H(j\omega)|^2 \approx A_0$  and  $|H(j\omega)|^2 \leq A_0$  in the pass band and  $|H(j\omega)|^2 \ll A_0$  in the stop band. These are the essential features of a lowpass characteristic that approximates the ideal lowpass characteristic.

### The Butterworth Lowpass Filter Characteristic

One of the simplest lowpass magnitude characteristics was first suggested by Butterworth. Because of its simplicity, it is often used when the filtering requirement is not too demanding. One of the simplest choices for  $F(\omega^2)$  in Eq. (1) is to make

$$F(\omega^2) = \omega^{2n} \quad (1)$$

where  $n$  is a positive integer. The magnitude-squared characteristic is

$$|H(j\omega)|^2 = \frac{1}{1 + \omega^{2n}} \tag{2}$$

A filter that satisfies (2.4) is known as the Butterworth filter of the  $n$ th order. It is clear that

$$|H(j\omega)|_{\max} = H(0) = 1 \tag{3}$$

for any  $n$ . As  $\omega$  increases,  $|H(j\omega)|$  decreases monotonically. Also, for any  $n$

$$|H(j1)|^2 = \frac{1}{2} \tag{4}$$

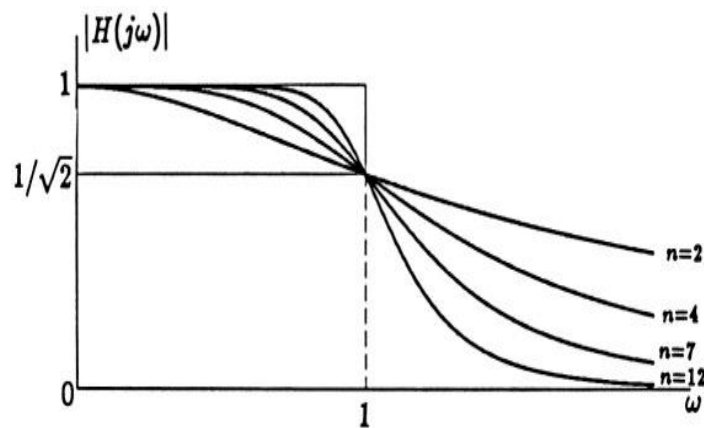
Hence any  $|H(j\omega)|$  given in (3) has the value  $1/\sqrt{2}$  at  $\omega = 1$ . The gain at this point is 3.0103 dB below the maximum gain. This point is commonly referred to as the 3-dB or half-power point.

If we apply the binomial expansion of  $|H(j\omega)|$ , we can write

$$|H(j\omega)| = (1 + \omega^{2n})^{-\frac{1}{2}} = ( 1 - \frac{1}{2} \omega^{2n} + \frac{3}{8} \omega^{4n} - \frac{5}{16} \omega^{6n} + \dots ) \tag{5}$$

In the vicinity of  $\omega = 0$ , the first  $2n-1$  derivatives of  $|H(j\omega)|$  are zero at  $\omega = 0$ . Since  $F(\omega^2)$  is of degree  $2n$  in  $\omega$  and we have made shows that we have made  $|H(j0)| = 1$ , Eq. (6) shows that we have made the  $|H(j\omega)|$  curve as flat as possible at  $\omega = 0$ . This characteristic is often referred to as the maximally flat magnitude characteristic. Hence, in the range  $0 < \omega < 1$ , the higher  $n$  is, the flatter the characteristic is at the origin, and it approaches the ideal lowpass characteristic.

For  $\omega > 1$ , the higher  $n$  is, the faster  $\omega^{2n}$  increases and the faster  $|H(j\omega)|$  decreases as  $\omega$  is increased. Figure 1 shows the  $|H(j\omega)|$  characteristics for several values of  $n$ .



**Figure 1** Normalized Butterworth magnitude characteristic for several value of  $n$

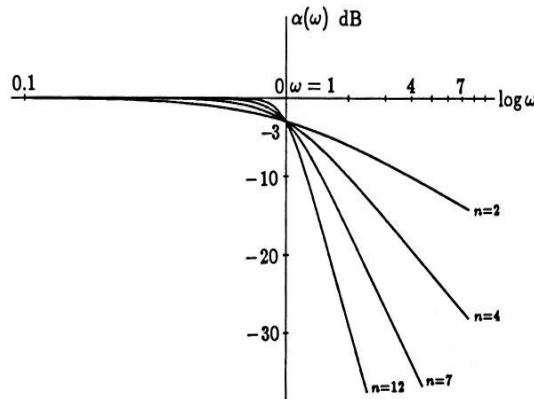
For  $\omega \gg 1$ , the Butterworth magnitude characteristic may be approximated

by

$$|H(j\omega)|^2 \approx \frac{1}{\omega^{2n}} \tag{6}$$

$$\alpha \approx -10\log(\omega^{2n}) = -20\log \omega \tag{7}$$

Hence the gain decreases at the rate of 20n dB/decade. The magnitude characteristics of Figure 1 plotted as dB versus log  $\omega$  - the Bode plots -are shown in Figure 2.



**Figure 2** The bode plot of the Butterworth magnitude characteristic of Figure 1

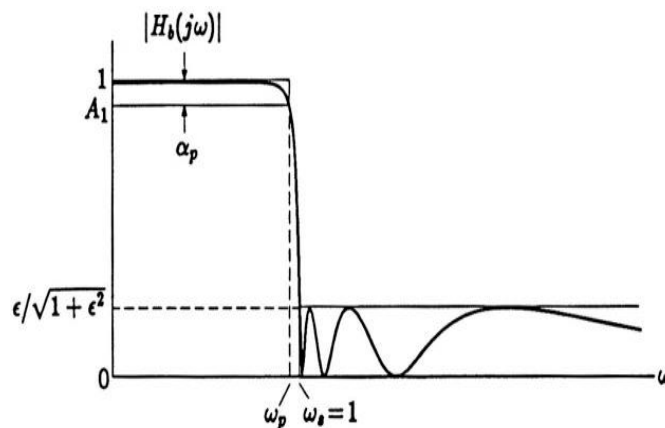
### The Chebyshev-II Lowpass Filter Characteristic

If we replace  $\omega$  by  $1/\omega$ , we get another magnitude-squared characteristic

$$|H_a(j\omega)|^2 = \frac{1}{1 + \epsilon^2 C_n^2(1/\omega)} \tag{9}$$

$$|H_b(j\omega)|^2 = 1 - |H_a(j\omega)|^2 \tag{10}$$

A lowpass characteristic  $|H_b(j\omega)|$  with equal-ripple variation in the stop band is obtained. The stop band is now the range  $1 < \omega < \infty$  and the pass band will be located somewhere in the range  $0 \leq \omega < 1$ . An example for  $n = 4$  is shown in Figure 3 This characteristic is known as the inverse Chebyshev characteristic or Chebyshev-II characteristic .



**Figure 3**The low pass ‘inverse Chebyshev’/ ‘Chebyshev-II’ characteristic

### Simulation Results:

The proposed design techniques of iir filter are discussed in this section. These techniques are analyzed for getting the noise power value through MATLAB R-2012.

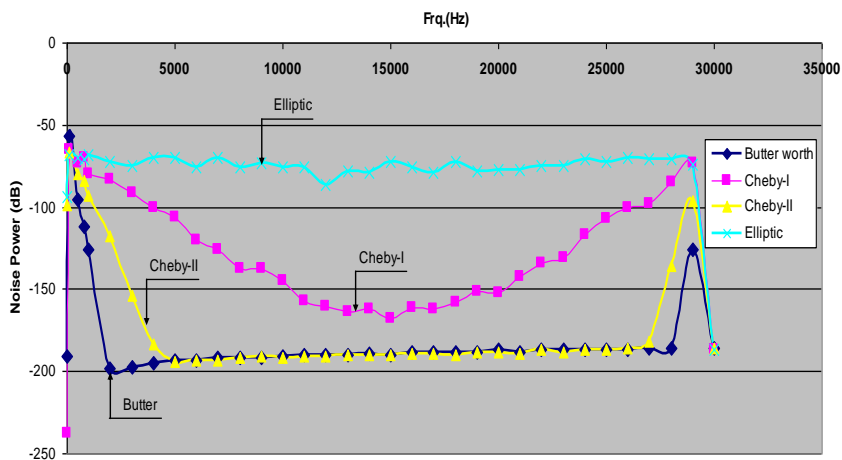
Here, all the 34 points observation is analyzed to find the amount of noise at various frequency points. As mentioned in Table 1 that, these 34 points observations is taken at various frequencies from 10 Hz to 30 kHz. The observation is average round-off noise power (in dB) that is obtained by various design method i.e. Butterworth, Chebyshev I & II and Elliptic filter. The data obtained in Table 1 is taken from Appendix-B, which is plotted in Figure 4

Since, the value of all round-off noise power in Table 1 is in negative, and then it means that higher the absolute value of noise power is having lower noise power. This is also obvious from Figure 4 that the lowest noise power value is far away from x-axis and towards the negative y-axis.

Table 1 Noise power at 34 points of frequency

Sl.No.	Frq. (Hz.)	Average Round-off Noise Power (dB) (Default arrangement of SOS)			
		Butterworth	Cheby-I	Cheby-II	Elliptic
1	10	-191	-238	-99	-94
2	100	-57	-65	-67	-68
3	500	-95	-73	-80	-70
4	800	-112	-70	-84	-71
5	1000	-126	-80	-93	-68
6	2000	-198	-83	-118	-72
7	3000	-197	-91	-154	-75
8	4000	-195	-100	-183	-70
9	5000	-193	-106	-194	-70
10	6000	-193	-120	-193	-76
11	7000	-192	-126	-193	-70
12	8000	-192	-137	-192	-76
13	9000	-192	-137	-191	-73
14	10000	-191	-145	-192	-76
15	11000	-190	-157	-191	-76
16	12000	-190	-160	-191	-86
17	13000	-190	-164	-190	-78
18	14000	-189	-162	-190	-79
19	15000	-190	-168	-190	-72

20	16000	-188	-161	-189	-76
21	17000	-188	-162	-189	-79
22	18000	-188	-158	-190	-72
23	19000	-188	-151	-188	-78
24	20000	-187	-152	-188	-77
25	21000	-188	-142	-189	-77
26	22000	-187	-134	-187	-75
27	23000	-187	-131	-188	-75
28	24000	-187	-117	-187	-71
29	25000	-187	-107	-187	-72
30	26000	-187	-100	-186	-70
31	27000	-186	-98	-182	-71
32	28000	-186	-85	-136	-71
33	29000	-126	-73	-96	-74
34	30000	-186	-187	-186	-187



**Figure 0** Frequency Vs. Average Round-off Noise Power (dB) due to default arrangement of SOS

**Conclusion**

Since, pass band of Butterworth and Chebyshev-II filter is similar type so, their noise response in Figure 4 is almost similar. Chebyshev-I filter has ripple in pass band so, it has responded higher noise while Elliptic filter has the highest noise in whole frequency band of 10 Hz to 30 kHz, as depicted in Figure 4. Since, the analysis is based on the finite precision arithmetic. If word length of storage device is increased, the result will be improved.

When the second-order section of the transfer function is rearranged, it is found that the noise power is changed. This happens because a new pair of pole and zero is formed at every selection of arrangement and delivers the noise power accordingly. In spite of this, if we expand the polynomial of the numerator and denominator in the variable  $z^{-1}$ , we will find a worse filter response. That is why, the optimum selection of a second-order section is necessary to obtain an optimum value of noise power. This explanation is based on Butterworth and Chebyshev-II filters. For Butterworth as a case here, at a lower frequency of 10 Hz, the amount of small noise is -191 dB, but as frequency increases from 10 Hz to 100 Hz, the noise increases and reaches a maximum value of -57 dB. Now, when frequency increases from 100 Hz to 2 kHz, it obtains a minimum value of -198 dB noise. When

frequency increases from 2 kHz to 28 kHz, noise power remains almost constant, and it varies from -198 dB to -186 dB. There is a sudden increase to -126 dB of noise power when frequency changes from 28 kHz to 29 kHz, and then decreases to -186 dB when frequency reached to 30 kHz. The similar type of variations also occurs for Chebyshev-II filter. The outcome of Chebyshev-I filter is like to above, but having small constant noise power frequency band. The value of noise power is higher in comparison to Butterworth & Chebyshev-II filter. The outcome of elliptic filter is the worst. It has almost constant noise power frequency band but, the highest value of noise. When other SOS arrangement methods are applied for reduction of reference noise, it comes to know that there is a small change in Butterworth and Chebyshev-II methods. These changes are occurring in both direction i.e. positive and negative direction of reference line. In case of Butterworth, it generally changes from 3 dB to -4 dB, and in case of Chebyshev-II it is also almost same i.e. from 3 dB to -4 dB. In case of Chebyshev-I and Elliptic, variation is almost double of above and it varies from 8 dB to -6 dB.

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