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Optimization of Fuzzy Inventory Model for Allowable Shortage

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Abstract: In this paper on fuzzy inventory model with allowable shortage has been considered in a fuzzy environment using the trapezoidal fuzzy numbers. Our aim is to consider the fuzzy optimal total cost and fuzzy optimal large size for the proposed inventory model. Holding cost, ordering cost, shortage cost and demand are taken as in terms of trapezoidal fuzzy numbers. A relevant numerical example is also included to justify the proposed notion.

Keywords: Trapezoidal fuzzy numbers, fuzzy inventory model, fuzzy optimal total cost, fuzzy optimal order quantity, allowable shortage.

I. INTRODUCTION

Inventory model was developed by Harris [4] and Wilson [11] aroused interest in the EOQ model in academics and industries. In 1970, Zadeh [12] proposed some strategies for decision making in fuzzy environment. Jain worked on decision making in the presence of fuzzy decision making models. Thus, EOQ model serves as a useful approximation to many real life problems. Urgeletti treated EOQ model in fuzzy sense and used triangular fuzzy number. Chan [1] and Wang [1] used trapezoidal fuzzy number to fuzzify the order cost, inventory cost and back order cost in the total cost of inventory model without back order. Further, in a series of papers, Yao [13] considered the fuzzified problems for the inventory with or without back order models. Kacpryzk [6] discussed some long-term inventory policy making through the fuzzy decision making models. Later Hadley [3] analyzed many inventory systems. Parvathi [7] and Gajalakshmi [7] investigate a deterministic inventory model with allowable shortage. Stephen Dinagar [9] and Rajesh Kannan [9] analysed on fuzzy inventory model with allowable shortage. In 2012, Chih Hsun Hsieh [2] analysed optimization of fuzzy production inventory models.

In this paper, a new fuzzy number called Trapezoidal fuzzy numbers, is utilized in developing the idea of inventory model though Rajarajesvari [8] proposed the above said fuzzy number without any restriction of parameter.

The main aim of this paper is to estimate the fuzzy optimal order quantity and fuzzy optimal total cost of an inventory system under study due to irregularities or physical properties of the parameter variables. For this situation, apply fuzzy concepts, shortage is allowed and it is completely backlogged.

In section 2 of this paper, some basic definitions and operations on trapezoidal fuzzy numbers are presented. Section 3 describes in brief the notions and assumptions used in the developed fuzzy inventory model. Formulation and analysis of the inventory model are in fuzzy sense and algorithm is presented. In section 4, a numerical example is given to illustrate the model. In section 5, the concluding remarks are also given.

II. DEFINITIONS AND METHODOLOGY

2.1 Fuzzy Set:

A fuzzy set $\tilde{A}$ on the given universal set X is a set of ordered pairs $\tilde{A} = \left\{ \left(x, \mu_{\tilde{A}}(x) \right) : x \in X \right\}$, where $\mu_{\tilde{A}} : X \to [0,1]$ is called membership function. The $\alpha -$ cut of $\tilde{A}$ is defined by $A_\alpha = \left\{ x : \mu_A(x) = \alpha, \alpha \geq 0 \right\}$. If $\mathbb{R}$ is the real line, then a fuzzy number is a fuzzy set $\tilde{A}$ with membership function $\mu_{\tilde{A}} : X \to [0,1]$ has the following properties:

i. $\tilde{A}$ is normal, (i.e.) there exists $x \in \mathbb{R}$ such that $\mu_A(x) = 1$.

ii. $\tilde{A}$ is piece –wise continuous.

iii. Sup $(\tilde{A}) = Cl \left\{ x \in \mathbb{R} : \mu_A(x) > 0 \right\}$ where Cl represents the closure of a set.

iv. $\tilde{A}$ is a convex fuzzy set.

2.2 Trapezoidal fuzzy number:

A trapezoidal fuzzy number $\tilde{A} = (a,b,c,d)$ is represented with membership function $\mu_{\tilde{A}}$ a

$$\mu_{\tilde{A}}(x) = \begin{cases} L(x) = \dfrac{x-a}{b-a}, a \leq x \leq b \\ 1, b \leq x \leq c \\ R(x) = \dfrac{d-x}{d-c}, c \leq x \leq d \\ 0, otherwise \end{cases}$$

2.3 Fuzzy set in LR-Form:

A fuzzy set is called in LR-Form, if there exist reference functions L (for left), R(for right) and scalars m>0 and n>0 with membership function.

$$\mu_{\tilde{A}}(x) = \begin{cases} L\left(\dfrac{\sigma - x}{m} \right), x \leq \sigma \\ 1, \sigma \leq x \leq \gamma \\ R\left(\dfrac{x-\gamma}{n} \right), x \geq \gamma \end{cases}$$

where σ is a real number called the mean value of $\tilde{A}$, m and n are called the left and right spreads respectively. The functions L and R map from $R^{+} \rightarrow [0,1]$ and are decreasing. An LR- type fuzzy number can be represented as $\tilde{A} = (\sigma, \gamma, m, n)_{LR}$.

2.4 Arithmetic Operations:

i. The addition of $\tilde{A}_1$ and $\tilde{A}_2$ is $\tilde{A}_1(+)\tilde{A}_2 = \left(a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4\right)$

ii. The subtraction of $\tilde{A}_1$ and $\tilde{A}_2$ is $\tilde{A}_1(-)\tilde{A}_2 = \left(a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1\right)$

iii. The multiplication of $\tilde{A}_1$ and $\tilde{A}_2$ is

$$\tilde{A}_1(\times)\tilde{A}_2 = \left(\frac{a_1}{4}\sigma_b, \frac{a_2}{4}\sigma_b, \frac{a_3}{4}\sigma_b, \frac{a_4}{4}\sigma_b\right), \sigma_b = \left(b_1 + b_2 + b_3 + b_4\right)$$

iv. The division of $\tilde{A}_1$ and $\tilde{A}_2$ is $\tilde{A}_1(\div)\tilde{A}_2 = \left(\frac{4a_1}{\sigma_b}, \frac{4a_2}{\sigma_b}, \frac{4a_3}{\sigma_b}, \frac{4a_4}{\sigma_b}\right)$ if

$\sigma_b \neq 0, \sigma_b = \left(b_1 + b_2 + b_3 + b_4\right)$

v. If $k \neq 0$ is a scalar $k\tilde{A}$ is defined as $k\tilde{A} = \begin{cases} \left(ka_1, ka_2, ka_3, ka_4\right), k > 0 \\ \left(ka_4, ka_3, ka_2, ka_1\right), k < 0 \end{cases}$

vi. $\sqrt{\tilde{A}} = \sqrt{a_1, a_2, a_3, a_4} = \left(\sqrt{a_1}, \sqrt{a_2}, \sqrt{a_3}, \sqrt{a_4}\right)$ where a_1, a_2, a_3, a_4 are non zero positive real numbers.

2.5 Methodology:

In Chih and Hsieh [2] introduced Graded Mean Integration Representation method based on the integral value of graded mean h-level of generalized fuzzy number for defuzzifying generalized fuzzy number. First illustrate generalized fuzzy number as follows:

Suppose $\tilde{A}$ is a generalized fuzzy number, it is described as any fuzzy subset of the real line $\mathbb{R}$, whose membership function $\mu_{\tilde{A}}$ satisfies the following conditions:

i. $\mu_{\tilde{A}}(x)$ is a continuous mapping from $\mathbb{R}$ to the closed interval [0,1]

ii. $\mu_{\tilde{A}}(x) = 0, -\infty < x \leq a_1$

iii. $\mu_{\tilde{A}}(x) = L(x)$ is strictly increasing on $[a_1, a_2]$

iv. $\mu_{\tilde{A}}(x) = w_A, a_2 \leq x \leq a_3$

v. $\mu_{\tilde{A}}(x) = R(x)$ is strictly decreasing on $[a_3, a_4]$

vi. $\mu_{\tilde{A}}(x) = 0, a_4 \leq x < \infty$ where $0 < w_A \leq 1$, and a_1, a_2, a_3, a_4 are real numbers.

Also this type of generalized fuzzy number is denoted as $\tilde{A} = (a_1, a_2, a_3, a_4; w_A)_{LR}$. When $w_A = 1$, it can be simplified as $\tilde{A} = (a_1, a_2, a_3, a_4)_{LR}$. Second, by Graded Mean Integration Representation method L^{-1} and R^{-1} are the inverse functions of L and R respectively and the graded mean h-level value of generalized fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4; w_A)_{LR}$ is $h\left(L^{-1}(h) + R^{-1}(h)\right)/2$. Then the Graded Mean Integration Representation of $\tilde{A}$ is P($\tilde{A}$) with grade $w_{A,}$ where $P(\tilde{A}) = \int_0^{w_A} h\left(\frac{L^{-1}(h) + R^{-1}(h)}{2}\right)dh \Big/ \int_0^{w_A} hdh$ with $0 < h \le w_A$ and $0 < w_A \le 1$.

Throughout this paper, only use popular trapezoidal fuzzy number as the type of all fuzzy parameters in our proposed fuzzy production inventory models. Let $\tilde{B}$ be a trapezoidal fuzzy number and is denoted as $\tilde{B} = (b_1, b_2, b_3, b_4)$, then we can get the Graded Mean Integration Representation of $\tilde{B}$ by the above formula as

$$P(\tilde{B}) = \int_0^1 h\left(\frac{b_1 + b_4 + (b_2 - b_1 - b_4 + b_3)h}{2}\right)dh \Big/ \int_0^1 hdh = \frac{b_1 + 2b_2 + 2b_3 + b_4}{6}$$

III. FUZZY INVENTORY MODEL

3.1 Symbols used:

C_h : Holding Cost per unit quantity per unit time

C_a : Set up cost or ordering cost per order

C_s : Shortage cost per unit quantity

D_t : Total demand over the planning time period [0,T]

Q: Order quantity per cycle

T_c: Total cost for the period [0,T]

$F(Q)^*$: Minimum total cost for [0,T]

Q_d^*: Optimal order quantity

$\tilde{C}_h$: Fuzzy holding cost per unit quantity per unit time

$\tilde{C}_a$: Fuzzy set up cost or ordering cost per order

T: Length of the plan

$\tilde{C}_s$: Fuzzy shortage cost per unit quantity

$\tilde{D}_t$: Fuzzy total demand over the planning time period [0,T]

$\tilde{Q}$: Fuzzy order quantity per cycle

$\tilde{T}_c$: Fuzzy total cost for the period [0,T]

$\tilde{F}(Q)^*$: Minimum fuzzy total cost for [0,T]

$\tilde{Q}_d^*$: Fuzzy optimal order quantity

3.2 Assumptions:

i. Shortage cost is fuzzy in nature.

ii. Total demand is fuzzy in nature.

iii. Time plan is constant.

iv. Holding cost and ordering cost are fuzzy in nature.

3.3 Formulation and Analysis of the model:

Consider the model with allowable shortage in fuzzy total demand with time period [0,T], fuzzy ordering cost per order, fuzzy carrying cost or holding cost per unit quantity per unit time and fuzzy shortage cost per unit quantity are in fuzzy sense. Our aim is to determine the fuzzy optimal total cost and fuzzy optimal order quantity for the proposed fuzzy inventory model. Now, fuzzifying the total cost is given by,

$$\tilde{T}_c = \frac{\tilde{H}T\tilde{Q}\tilde{S}^2}{2\left(\tilde{H}T+\tilde{S}\right)^2} + \frac{\tilde{S}\tilde{Q}\left(1-\frac{\tilde{S}}{\tilde{H}T+\tilde{S}}\right)^2}{2} + \frac{\tilde{A}\tilde{D}}{\tilde{Q}}$$

Our focus is to obtain fuzzy total cost and the large size in terms of trapezoidal fuzzy numbers by using simple techniques. If $\tilde{C}_h, \tilde{C}_a, \tilde{C}_s, \tilde{D}_t$ denote the trapezoidal fuzzy numbers defined by,

$$\tilde{C}_h = (C_{h_1}, C_{h_2}, C_{h_3}, C_{h_4}), \tilde{D}_t = (D_{t_1}, D_{t_2}, D_{t_3}, D_{t_4}), \tilde{C}_a = (C_{a_1}, C_{a_2}, C_{a_3}, C_{a_4}), \tilde{C}_s = (C_{s_1}, C_{s_2}, C_{s_3}, C_{s_4})$$

$$\tilde{T}_c = \left\{ \frac{\tilde{C}_h T\tilde{Q}(\tilde{C}_s)^2}{2(\tilde{C}_h T+\tilde{C}_s)^2} + \frac{\tilde{C}_s\tilde{Q}\left(1-\frac{\tilde{C}_s}{\tilde{C}_h T+\tilde{C}_s}\right)^2}{2} + \frac{\tilde{C}_a\tilde{D}_t}{\tilde{Q}} \right\}$$

By using new arithmetic operations and simplifying, we get

$$\tilde{T}_c = \begin{cases} \frac{TQ}{4}C_{s_1}\frac{\partial_a\partial_b}{\partial_c} + \frac{Q}{8}\left(\frac{TC_{h_1}+C_{S_1}-C_{s_4}}{\partial_d}\right)\partial_a\partial_e + \frac{1}{4Q}D_{t_1}\partial_f, \\ \frac{TQ}{4}C_{s_2}\frac{\partial_a\partial_b}{\partial_c} + \frac{Q}{8}\left(\frac{TC_{h_2}+C_{S_2}-C_{s_3}}{\partial_d}\right)\partial_a\partial_e + \frac{1}{4Q}D_{t_2}\partial_f, \\ \frac{TQ}{4}C_{s_3}\frac{\partial_a\partial_b}{\partial_c} + \frac{Q}{8}\left(\frac{TC_{h_3}+C_{S_3}-C_{s_2}}{\partial_d}\right)\partial_a\partial_e + \frac{1}{4Q}D_{t_3}\partial_f, \\ \frac{TQ}{4}C_{s_4}\frac{\partial_a\partial_b}{\partial_c} + \frac{Q}{8}\left(\frac{TC_{h_4}+C_{S_4}-C_{s_1}}{\partial_d}\right)\partial_a\partial_e + \frac{1}{4Q}D_{t_4}\partial_f \end{cases}$$

$\tilde{T}_c = \left(a_1, a_2, a_3, a_4\right) = F(\tilde{Q})$ where $\partial_a = \left(C_{s_1}+C_{s_2}+C_{s_3}+C_{s_4}\right)$, $\partial_b = \left(C_{h_1}+C_{h_2}+C_{h_3}+C_{h_4}\right)$,

$$\partial_c = \begin{bmatrix} \frac{TC_{h_1}+C_{s_1}}{2}\left(TC_{h_1}+C_{s_1}+TC_{h_2}+C_{s_2}+TC_{h_3}+C_{s_3}+TC_{h_4}+C_{s_4}\right)+ \\ \frac{TC_{h_2}+C_{s_2}}{2}\left(TC_{h_1}+C_{s_1}+TC_{h_2}+C_{s_2}+TC_{h_3}+C_{s_3}+TC_{h_4}+C_{s_4}\right)+ \\ \frac{TC_{h_3}+C_{s_3}}{2}\left(TC_{h_1}+C_{s_1}+TC_{h_2}+C_{s_2}+TC_{h_3}+C_{s_3}+TC_{h_4}+C_{s_4}\right)+ \\ \frac{TC_{h_4}+C_{s_4}}{2}\left(TC_{h_1}+C_{s_1}+TC_{h_2}+C_{s_2}+TC_{h_3}+C_{s_3}+TC_{h_4}+C_{s_4}\right) \end{bmatrix},$$

$$\partial_d = (TC_{h_1}+C_{s_1}+TC_{h_2}+C_{s_2}+TC_{h_3}+C_{s_3}+TC_{h_4}+C_{s_4}),$$

$$\partial_e = \begin{pmatrix} \frac{4\left(TC_{h_1}+C_{s_1}-C_{s_4}\right)}{\left(TC_{h_1}+C_{s_1}+TC_{h_2}+C_{s_2}+TC_{h_3}+C_{s_3}+TC_{h_4}+C_{s_4}\right)}+ \\ \frac{4\left(TC_{h_2}+C_{s_2}-C_{s_3}\right)}{\left(TC_{h_1}+C_{s_1}+TC_{h_2}+C_{s_2}+TC_{h_3}+C_{s_3}+TC_{h_4}+C_{s_4}\right)}+ \\ \frac{4\left(TC_{h_3}+C_{s_3}-C_{s_2}\right)}{\left(TC_{h_1}+C_{s_1}+TC_{h_2}+C_{s_2}+TC_{h_3}+C_{s_3}+TC_{h_4}+C_{s_4}\right)}+ \\ \frac{4\left(TC_{h_4}+C_{s_4}-C_{s_1}\right)}{\left(TC_{h_1}+C_{s_1}+TC_{h_2}+C_{s_2}+TC_{h_3}+C_{s_3}+TC_{h_4}+C_{s_4}\right)} \end{pmatrix},$$

$\partial_f = \left(C_{a_1}+C_{a_2}+C_{a_3}+C_{a_4}\right)$, $\partial_g = (Q_{d_1}+Q_{d_2}+Q_{d_3}+Q_{d_4})$. The fuzzy optimal order quantity $\tilde{Q}_d^*$ which minimizes the total inventory cost $\tilde{T}_c = F(\tilde{Q})$ is obtained as the solution of the first order fuzzy differential equation $\frac{d}{d\tilde{Q}}(\tilde{T}_c) = \tilde{0}$ and it is found as, $\tilde{Q}_d^* = \sqrt{\begin{matrix} \frac{2D_{t_1}\partial_f}{\partial_a}+\frac{2D_{t_1}\partial_f}{T\partial_b}, \frac{2D_{t_2}\partial_f}{\partial_a}+\frac{2D_{t_2}\partial_f}{T\partial_b}, \\ \frac{2D_{t_3}\partial_f}{\partial_a}+\frac{2D_{t_3}\partial_f}{T\partial_b}, \frac{2D_{t_4}\partial_f}{\partial_a}+\frac{2D_{t_4}\partial_f}{T\partial_b} \end{matrix}}$

$\tilde{Q}_d^* = (Q_{d_1}, Q_{d_2}, Q_{d_3}, Q_{d_4})$ by using arithmetic operation.

Also, $\tilde{Q} = \tilde{Q}_d^*$ we have $\frac{d^2F(\tilde{Q})}{d\tilde{Q}^2} > 0$. This shows that $F(\tilde{Q})$ is minimum at $\tilde{Q} = \tilde{Q}_d^*$ and

$$F(Q^*) = \begin{cases} \dfrac{\frac{T}{16} C_{s_1} \partial_a \partial_b \partial_g}{\partial_c} + \dfrac{1}{32}\left(\dfrac{(TC_{h_1} + C_{s_1} - C_{s_4})\partial_a \partial_g}{\partial_d} \partial_e \right) + \dfrac{D_{t_1} \partial_f}{\partial_g}, \\ \dfrac{\frac{T}{16} C_{s_2} \partial_a \partial_b \partial_g}{\partial_c} + \dfrac{1}{32}\left(\dfrac{(TC_{h_2} + C_{s_2} - C_{s_3})\partial_a \partial_g}{\partial_d} \partial_e \right) + \dfrac{D_{t_2} \partial_f}{\partial_g}, \\ \dfrac{\frac{T}{16} C_{s_3} \partial_a \partial_b \partial_g}{\partial_c} + \dfrac{1}{32}\left(\dfrac{(TC_{h_3} + C_{s_3} - C_{s_2})\partial_a \partial_g}{\partial_d} \partial_e \right) + \dfrac{D_{t_3} \partial_f}{\partial_g}, \\ \dfrac{\frac{T}{16} C_{s_4} \partial_a \partial_b \partial_g}{\partial_c} + \dfrac{1}{32}\left(\dfrac{(TC_{h_4} + C_{s_4} - C_{s_1})\partial_a \partial_g}{\partial_d} \partial_e \right) + \dfrac{D_{t_4} \partial_f}{\partial_g} \end{cases}$$

3.4 Algorithm for finding fuzzy optimal total cost and fuzzy optimal order quantity:

Step 1:

Calculate the model fuzzy total cost for the fuzzy values of $\tilde{C}_h, \tilde{C}_a, \tilde{C}_s, \tilde{D}_t$

Step 2:

Determine fuzzy total cost using new arithmetic operations fuzzy holding cost, fuzzy ordering cost, fuzzy shortage cost and fuzzy demand taken in terms of trapezoidal fuzzy numbers.

Step 3:

Find the fuzzy optimal order quantity which can be obtained by putting the first derivative of $F(\tilde{Q})$ equal to zero and second derivative is positive at $\tilde{Q} = \tilde{Q}_d^*$.

IV. NUMERICAL EXAMPLE

Tata manufacturing produces commercial mobile units in batches. The firm's estimated demand for the year is greater than or less than 500 units. The setup cost is about \$20. Length of the plan is 6 days and the holding cost per unit quantity is 12. Shortage cost 6 per unit. Calculate the minimum fuzzy total cost and fuzzy optimal order quantity?

Solution:

$$\partial_a = (C_{s_1} + C_{s_2} + C_{s_3} + C_{s_4}) = 24, \quad \partial_b = (C_{h_1} + C_{h_2} + C_{h_3} + C_{h_4}) = 48$$

$$\partial_c = \begin{bmatrix} \dfrac{TC_{h_1} + C_{s_1}}{2}\left(TC_{h_1} + C_{s_1} + TC_{h_2} + C_{s_2} + TC_{h_3} + C_{s_3} + TC_{h_4} + C_{s_4}\right) + \\ \dfrac{TC_{h_2} + C_{s_2}}{2}\left(TC_{h_1} + C_{s_1} + TC_{h_2} + C_{s_2} + TC_{h_3} + C_{s_3} + TC_{h_4} + C_{s_4}\right) + \\ \dfrac{TC_{h_3} + C_{s_3}}{2}\left(TC_{h_1} + C_{s_1} + TC_{h_2} + C_{s_2} + TC_{h_3} + C_{s_3} + TC_{h_4} + C_{s_4}\right) + \\ \dfrac{TC_{h_4} + C_{s_4}}{2}\left(TC_{h_1} + C_{s_1} + TC_{h_2} + C_{s_2} + TC_{h_3} + C_{s_3} + TC_{h_4} + C_{s_4}\right) \end{bmatrix}$$

$$\partial_C = 48672$$

$$\partial_d = (TC_{h_1} + C_{s_1} + TC_{h_2} + C_{s_2} + TC_{h_3} + C_{s_3} + TC_{h_4} + C_{s_4}) = 312$$

$$\partial_e = \left(\begin{array}{l} \dfrac{4(TC_{h_1} + C_{s_1} - C_{s_4})}{(TC_{h_1} + C_{s_1} + TC_{h_2} + C_{s_2} + TC_{h_3} + C_{s_3} + TC_{h_4} + C_{s_4})} + \\ \dfrac{4(TC_{h_2} + C_{s_2} - C_{s_3})}{(TC_{h_1} + C_{s_1} + TC_{h_2} + C_{s_2} + TC_{h_3} + C_{s_3} + TC_{h_4} + C_{s_4})} + \\ \dfrac{4(TC_{h_3} + C_{s_3} - C_{s_2})}{(TC_{h_1} + C_{s_1} + TC_{h_2} + C_{s_2} + TC_{h_3} + C_{s_3} + TC_{h_4} + C_{s_4})} + \\ \dfrac{4(TC_{h_4} + C_{s_4} - C_{s_1})}{(TC_{h_1} + C_{s_1} + TC_{h_2} + C_{s_2} + TC_{h_3} + C_{s_3} + TC_{h_4} + C_{s_4})} \end{array} \right)$$

∂_e **= 3.692**

$$\partial_f = \left(C_{a_1} + C_{a_2} + C_{a_3} + C_{a_4}\right) = 80, \ \partial_g = (Q_{d_1} + Q_{d_2} + Q_{d_3} + Q_{d_4}) = 233.5$$

$$\tilde{Q}_d^* = \sqrt{\begin{array}{l} \dfrac{2D_{t_1}\partial_f}{\partial_a} + \dfrac{2D_{t_1}\partial_f}{T\partial_b}, \dfrac{2D_{t_2}\partial_f}{\partial_a} + \dfrac{2D_{t_2}\partial_f}{T\partial_b}, \\ \dfrac{2D_{t_3}\partial_f}{\partial_a} + \dfrac{2D_{t_3}\partial_f}{T\partial_b}, \dfrac{2D_{t_4}\partial_f}{\partial_a} + \dfrac{2D_{t_4}\partial_f}{T\partial_b} \end{array}}$$

$$\tilde{Q}_d^* = \left(38.005, 53.74, 65.82, 76.01\right)$$

$$F(\tilde{Q})^* = \begin{cases} \dfrac{\frac{T}{16} C_{s_1}\partial_a\partial_b\partial_g}{\partial_c} + \dfrac{1}{32}\left(\dfrac{(TC_{h_1} + C_{s_1} - C_{s_4})\partial_a\partial_g}{\partial_d}\partial_e \right) + \dfrac{D_{t_1}\partial_f}{\partial_g}, \\ \dfrac{\frac{T}{16} C_{s_2}\partial_a\partial_b\partial_g}{\partial_c} + \dfrac{1}{32}\left(\dfrac{(TC_{h_2} + C_{s_2} - C_{s_3})\partial_a\partial_g}{\partial_d}\partial_e \right) + \dfrac{D_{t_2}\partial_f}{\partial_g}, \\ \dfrac{\frac{T}{16} C_{s_3}\partial_a\partial_b\partial_g}{\partial_c} + \dfrac{1}{32}\left(\dfrac{(TC_{h_3} + C_{s_3} - C_{s_2})\partial_a\partial_g}{\partial_d}\partial_e \right) + \dfrac{D_{t_3}\partial_f}{\partial_g}, \\ \dfrac{\frac{T}{16} C_{s_4}\partial_a\partial_b\partial_g}{\partial_c} + \dfrac{1}{32}\left(\dfrac{(TC_{h_4} + C_{s_4} - C_{s_1})\partial_a\partial_g}{\partial_d}\partial_e \right) + \dfrac{D_{t_4}\partial_f}{\partial_g} \end{cases}$$

$$F(\tilde{Q})^* = \left(155.91, 280.90, 387.069, 512.061\right)$$

V. CONCLUSION

In this paper, fuzzy optimal order quantity and fuzzy optimal total cost is studied with the help of trapezoidal fuzzy number. To find various fuzzy optimal quantities, the demand, holding cost, ordering cost and shortage cost using trapezoidal fuzzy numbers have been used. New arithmetic operations of trapezoidal fuzzy numbers are proposed to get the required result. Hence, the fuzzy values are all closer to the crisp values of the real systems.

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