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RESEARCH ARTICLE

IMAGE COMPRESSION USING HIRARCHICAL LINEAR POLYNOMIAL CODING

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Abstract— In this paper a hierarchal modelling based is introduced for compressing images, it is based utilizing the layered representation along with the polynomial coding. The test results showed best performance of the hierarchal polynomial coding compared to the traditional polynomial coding.

Keywords— image compression, redundancy, modeling, hierarchal scheme, polynomial coding

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I. INTRODUCTION

In recent years, a dramatic increase in the amount of information available in the form of digital image data, it become necessary to solve the problems of storage and time issues by utilizing image compression of redundancy removal based. In general, Image compression techniques generally fall into two categories: lossless and lossy depending on the redundancy type exploited, where lossless also called information preserving or error free techniques, in which the image compressed without losing information that rearrange or reorder the image content, and are based on the utilization of statistical redundancy alone such as Huffman coding, Arithmetic coding and Lempel-Ziv algorithm, while lossy which remove content from the image, which degrades the compressed image quality, and are based on the utilization of psycho-visual redundancy, either solely or combined with statistical redundancy such as vector quantization, fractal, block truncation coding and JPEG [1], reviews of lossless and lossy techniques can be found in [2],[3],[4]-[7].

Modelling or a Mathematical Model is a simple description formula utilized efficiently in image compression problem to remove the correlation embedded between image pixel neighbours (spatial/interpixel redundancy). A compression system of modelled based, is generally composed of two parts; one corresponds to mathematical function (deterministic part) exploited to create an approximation modelled image that resemble the original image, and the second part corresponds to the error or residual (probabilistic part) as a difference between original and the approximated. For more details see [8], [9], [10]. Polynomial coding is modelling based technique exploited by a number of researchers as a tool to compress images [11], [12], [13]-[16]. The techniques characterized by simplicity of implementation, efficiency in reducing image information into small effective coefficients.

In this paper, the polynomial coding techniques adopted hierarchally to efficiently remove the dependency (correlation or redundancy) between neighbouring image pixels and between neighbouring coefficients. The rest of the paper organized as follows, section 2 discusses the proposed technique in more details; the result is given in section 3.

II. THE PROPOSED COMPRESSION SYSTEM

The main taken concerns in the proposed system are:

1- Polynomial coding of linear approximation model is exploited to compress image efficiently using the three coefficients ($a0, a_1$ *and* a_2) representation that remove the redundancy between the image itself.

2- The top-down layered or hierarchal scheme is adopted to remove the redundancy embedded within the coefficients to improve the compression ratio with preserving image quality.

The steps below illustrated the system implantation in more details; Figure (1) shows the basic steps clearly:

Step 1: Load the input uncompressed image I of size $N\times N$ that corresponds to layer$_0$ or the root of the hierarchal representation.

Step 2: Construct the first layer hierarchal representation corresponds to layer$_1$ using the linear polynomial coding techniques, such as:

1. Partition the input image I into non-overlapping blocks of fixed sized $n\times n$ (i.e., 4×4, 8×8).
2. Find the coefficients of the linear approximation model, using the equations below [17]:

$$a_0 = \frac{1}{n\times n}\sum_{i=0}^{n-1}\sum_{j=0}^{n-1} I(i,j) \ldots\ldots\ldots\ldots (1)$$

$$a_1 = \frac{\sum_{i=0}^{n-1}\sum_{j=0}^{n-1} I(i,j)\times(j-x_c)}{\sum_{i=0}^{n-1}\sum_{j=0}^{n-1}(j-x_c)^2} \ldots\ldots\ldots\ldots (2)$$

$$a_2 = \frac{\sum_{i=0}^{n-1}\sum_{j=0}^{n-1} I(i,j)\times(i-y_c)}{\sum_{i=0}^{n-1}\sum_{j=0}^{n-1}(i-y_c)^2} \ldots\ldots\ldots (3)$$

Where $I(i,j)$ is the original image block of size ($n\times n$) and

$$xc = yc = \frac{n-1}{2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Here the (j-x_c) and (i-y_c) corresponds to the variables of the polynomial that measure the distance of pixel coordinates to the block center (x_c, y_c). The a_0 coefficients represent the block mean, the a_1 coefficients and a_2 coefficients represent the ratio of sum pixel multiplied by the distance from the center to the squared distance in i and j coordinates respectively.

3. Quantized/dequantized the a_1 *and* a_2 computed coefficients above, using the uniform scalar quantizer.

$$a_1Q = round(\frac{a_1}{QS_{a1}}) \rightarrow a_1D = a_1Q \times QS_{a1} \ldots\ldots\ldots (5)$$

$$a_2Q = round(\frac{a_2}{QS_{a1}}) \rightarrow a_2D = a_1Q \times QS_{a1} \ldots\ldots\ldots (6)$$

One quantization step QS_{a1} is adopted for the a_1 and a_2 coefficients (the same quantization step used for both of them), for the quantized a_1Q, a_2Q /de-quantized a_1D, a_2D coefficients.

Step 3: Construct the second layer hierarchal representation corresponds to $layer_2$ from the previous layer coefficients ($layer_1$ coefficients), the a_0 corresponds to mean (average) of the image, the linear polynomial coding techniques utilized, as follows:

1. Partition the computed a_0 from $layer_1$ into non-overlapping blocks of fixed sized $n\times n$ (i.e., 4×4, 8×8), the size of a_0 is equal to $N/n\times N/n$.
2. Find the coefficients of a_0 of the linear approximation model, using the equations below:

$$a_{00} = \frac{1}{n\times n}\sum_{i=0}^{n-1}\sum_{j=0}^{n-1} a_0(i,j) \quad \ldots\ldots\ldots\ldots(7)$$

$$a_{01} = \frac{\sum_{i=0}^{n-1}\sum_{j=0}^{n-1} a_0(i,j)\times(j-x_c)}{\sum_{i=0}^{n-1}\sum_{j=0}^{n-1}(j-x_c)^2} \quad \ldots\ldots(8)$$

$$a_{02} = \frac{\sum_{i=0}^{n-1}\sum_{j=0}^{n-1} a_0(i,j)\times(i-y_c)}{\sum_{i=0}^{n-1}\sum_{j=0}^{n-1}(i-y_c)^2} \quad \ldots\ldots(9)$$

Where $a_0(i,j)$ is the mean of original image of block of size ($n\times n$) and

$$xc = yc = \frac{n-1}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots(10)$$

The a_{00}, a_{01} and a_{02} coefficients correspond to $layer_2$ constructed using the a_0 coefficients from $layer_1$ that regarded as an image.

3. Quantized/dequantized the a_{00}, a_{01} and a_{01} computed coefficients above, using the uniform scalar quantizer.

$$a_{00}Q = round\left(\frac{a_{00}}{QS_{a00}}\right) \rightarrow a_{00}D = a_{00}Q \times QS_{a00} \quad \ldots\ldots(11)$$

$$a_{01}Q = round\left(\frac{a_{01}}{QS_{a01}}\right) \rightarrow a_{01}D = a_{01}Q \times QS_{a01} \quad \ldots\ldots(12)$$

$$a_{02}Q = round\left(\frac{a_{02}}{QS_{a01}}\right) \rightarrow a_{02}D = a_{02}Q \times QS_{a01} \quad \ldots\ldots(13)$$

Two quantization steps QS_{a0}, QS_{a1} adopted one for the a_{00} coefficients, and one for a_{01} and a_{02} coefficients, for the quantized $a_{00}Q, a_{01}Q, a_{02}Q$ /dequantized $a_{00}D, a_{01}D, a_{02}D$ coefficients.

4. Determine the deterministic part (function formula) $\tilde{a}_0$ of mathematical linear model base using the dequantized coefficient and the variables.

$$\tilde{a}_0 = a_{00}D + a_{01}D(j-x_c) + a_{02}D(i-y_c) \quad \ldots\ldots\ldots\ldots\ldots\ldots(14)$$

5. Find the probabilistic part or error (residual) as a difference between the modelled approximated image $\tilde{a}_0$ and the original one a_0.

$$a_0E = \tilde{a}_0 - a_0 \quad \ldots\ldots\ldots\ldots(15)$$

6. Quantized/dequantized the error, using the uniform scalar quantizer.

$$a_0EQ = round(\frac{a_{0E}}{QS_{a0E}}) \rightarrow a_{0E}D = a_{0E}Q \times QS_{a0E} \ldots\ldots\ldots(16)$$

Where QS_{a0E} is the error quantization step for the quantized a_0EQ /dequantized $a_{0E}D$ coefficients.

Step 4: Build the approximated up layers from the subsequent layers, namely construct $layer_1$ from $layer_2$ and $layer_0$ from $layer_1$, such as:

1. Build the modeled approximated $\hat{a}_0$ corresponds to $layer_1$, using the two modeling parts, approximated $\tilde{a}_0$ and the error $a_{0E}D$.

$$\hat{a}_0 = \tilde{a}_0 + a_{0E}D \ldots\ldots\ldots\ldots\ldots\ldots(17)$$

2. Determine the deterministic part $\tilde{I}$ of mathematical linear model base using the dequantized coefficient of $layer_1$&$layer_2$, and the variables

$$\tilde{I} = \hat{a}0 + a_1D(j - x_c) + a_2D(i - y_c) \ldots\ldots\ldots\ldots\ldots\ldots(18)$$

3. Find the probabilistic part or error (residual) as a difference between the modelled approximated image $\tilde{I}$ and the original one *I*.

$$IE = \tilde{I} - I \ldots\ldots\ldots\ldots\ldots(19)$$

4. Quantized/dequantized the error *IE*, using the uniform scalar quantizer.

$$IEQ = round(\frac{IE}{QS_{IE}}) \rightarrow IED = IEQ \times QS_{IE} \ldots\ldots\ldots(20)$$

Where QS_{IE} is the error quantization step for the quantized IEQ /de-quantized IED coefficients.

5. Build the modeled approximated $\hat{I}$ corresponds to $layer_0$, using the two modeling parts, approximated $\tilde{I}$ and the error IED .

$$\hat{I} = \tilde{I} + IED \ldots\ldots\ldots\ldots\ldots\ldots\ldots(21)$$

6. Encode the $layer_2$ information of quantized coefficients ($a_{00}D, a_{01}D, a_{02}D$) and the quantized error ($a_{0E}D$) along with the $layer_0$ information of quantized coefficients (a_1D, a_2D) and the quantized error (*IE*) using LZW coding techniques.

The techniques, worked reversely from subsequent layers, to construct up layers, means using the coefficients (a_{00},a_{01},a_{02}) of $layer_2$ to construct approximated $layer_1$($\hat{a}_0$) and then using the $layer_1$ coefficients ($\hat{a}_0$,a_1,a_2) to construct the approximated image $\hat{I}$.

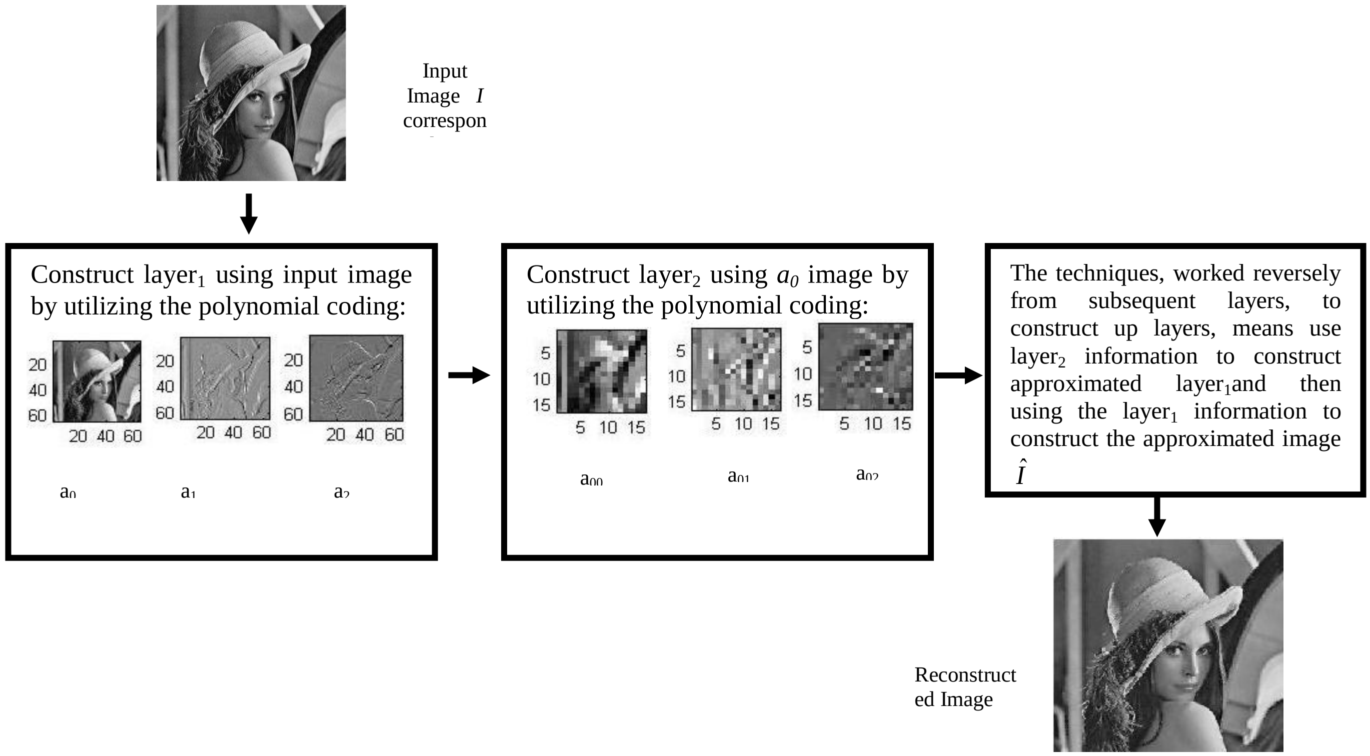

Fig. (1): The proposed hierarchal polynomial compression system in practical example.

Experimental Results

Experiments were done to compare the performance of the suggested the hierarchical polynomial coding with the traditional polynomial using a fixed block of size 4×4, with various quantization steps for errors (residual images) in $layer_1$ and $layer_2$, whereas the quantization steps for the coefficients adopted the as the identical for the both layers (i.e., use the same quantization step for the coefficients in $layer_1$ and $layer_2$ a_0,a_1,a_2, a_{00}, a_{01} and a_{02}). All the images used are standards (see Figure 2 for an overview) of 256 gray levels (8bits/pixel) of size 256×256.

The Compression ratio (ratio of original size to the compressed size in byte) and the Peak Signal to Noise Ratio (PSNR) adopted as an objective fidelity measure between the original image I and the decoded image $\hat{I}$ as in equation (22).

$$PSNR = 10\log_{10} \frac{255^2}{\frac{1}{N \times N}\sum_{x=0}^{N-1}\sum_{y=01}^{N-1}[\hat{I}(x,y) - I(x,y)]^2} \quad ....(22)$$

The experimental results are listed in tables (1) and (2) for traditional and hierarchal polynomial coding respectively, that showed that the performance improved using hierarchical polynomial coding techniques in terms of compression ratio about one and a half on average along due to the reduction of a_0 resolution (i.e., a_0 corresponds to mean of the image implicitly meaning overburden problem that consuming extra large number of bits) with the preserving the image quality.
Generally, two layers construction is sufficient to remove the redundancy, actually there's no need to extend the work to third layer where's no correlation embedded between $layer_2$ coefficients.
Lastly, there's a trade off between compression ratio and the quality affected by the quantization step and the block size, where for high quality image, low compression ratio achieved, that implicitly means small block size utilized with low quantization step, and vice versa, figure(3) shows an example of decoded images.

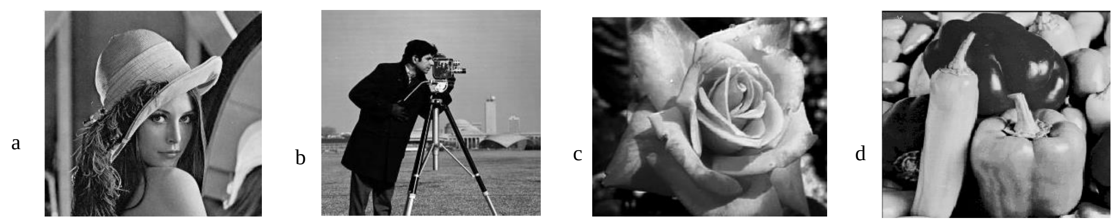

Fig. (2): Tested images (a) Lena, (b) Cameraman (c)Rose and (d) Paper, gray scale images of size 256×256.

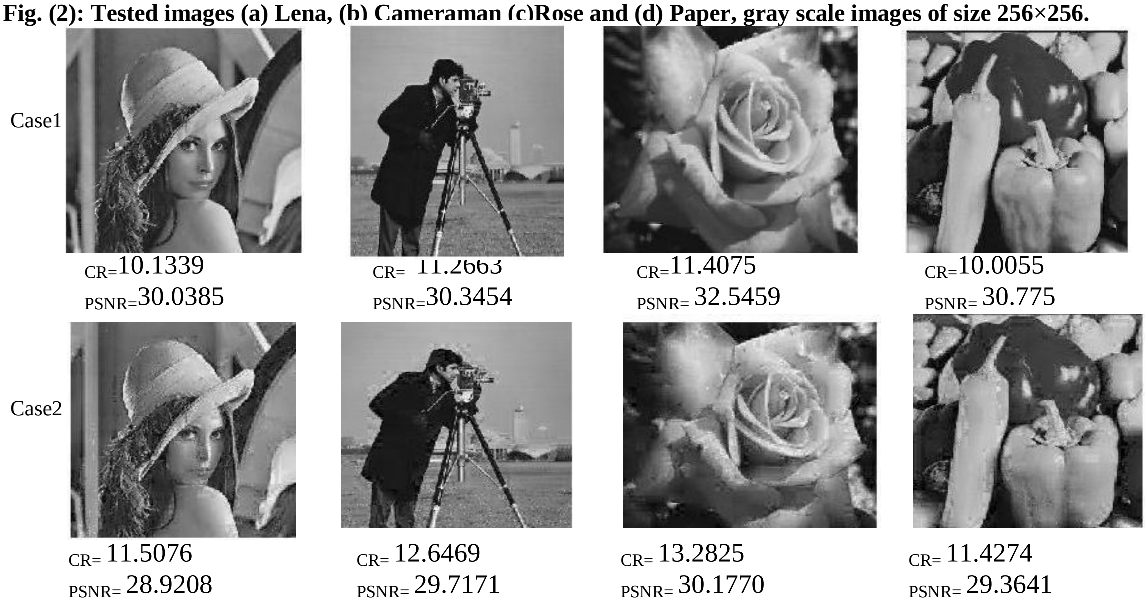

Fig. (3): Decoded image using the hierarchal polynomial coding, using quantization coefficients equals to 1 for both layers, quantization step of error layer1 error is equal to 50, with (Case1) quantization step of error layer2 is equal to 2 and (Case2) quantization step of error layer2 is equal to 20

Table 1: Traditional polynomial coding with quantization step equal to one for all the coefficients

Image	Qant. Error	CR	PSNR
Lena	2	3.0878	52.1956
	5	4.1387	45.0256
	10	5.1846	39.3291
	20	6.6366	34.9348
	30	7.4220	32.6981
	40	7.9727	31.1589
	50	8.3485	30.0512
Rose	2	3.5804	52.4918
	5	4.9257	45.5553
	10	6.1335	40.4278
	20	7.5502	36.4108
	30	8.3411	34.4462
	40	8.8407	33.3011
	50	9.1288	32.5719
Papper	2	3.2662	52.4004
	5	4.4628	45.4686
	10	5.5483	40.1211
	20	6.8725	35.7327
	30	7.6347	33.3989
	40	8.1261	31.8245
	50	8.4443	30.7833
Camera man	2	3.5112	52.2085
	5	4.9465	45.6249
	10	6.2049	41.0554
	20	7.4384	36.3415
	30	8.2456	33.5677
	40	8.8959	31.7294
	50	9.3905	30.3607

Table 2: Hierarchal polynomial coding with quantization step equal to one for all the coefficients in $layer_1$&$layer_2$ uses a selected case from the traditional polynomial coding when quantization of error is equal to 50.

Image	Qant. Error	CR	PSNR
Lena	2	3.0878	52.1956
	5	4.1387	45.0256
	10	5.1846	39.3291
	20	6.6366	34.9348
	30	7.4220	32.6981
	40	7.9727	31.1589
	50	8.3485	30.0512
Rose	2	3.5804	52.4918
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	20	7.5502	36.4108
	30	8.3411	34.4462
	40	8.8407	33.3011
	50	9.1288	32.5719
Papper	2	3.2662	52.4004
	5	4.4628	45.4686
	10	5.5483	40.1211
	20	6.8725	35.7327
	30	7.6347	33.3989
	40	8.1261	31.8245
	50	8.4443	30.7833
Camera man	2	3.5112	52.2085
	5	4.9465	45.6249
	10	6.2049	41.0554
	20	7.4384	36.3415
	30	8.2456	33.5677
	40	8.8959	31.7294
	50	9.3905	30.3607

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