Available Online at www.ijcsmc.com

International Journal of Computer Science and Mobile Computing

A Monthly Journal of Computer Science and Information Technology

ISSN 2320–088X
IMPACT FACTOR: 6.017

IJCSMC, Vol. 6, Issue. 1, January 2017, pg.182 – 189

The Edge Domination in Prime Square Dominating Graphs

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***Abstract*— This paper brings into view various findings related to edge domination numbers of prime square dominating graphs. Some classified prime square dominating graphs are chosen for this means of study and their edge domination numbers are resolved.**

***Keywords*— prime square dominating graph, edge dominating set, edge domination number, independent edge dominating set, independent edge domination number.**

I. INTRODUCTION

Graphs considered in this paper are all undirected, connected and simple graphs. Throughout this paper, the graph G = (V, E). Terms not defined here are used in the sense of Harary [1].

Mitchell and Hedetniemi [2] introduced the concept of edge domination. A subset X of E is called an edge dominating set of G if every edge not in X is adjacent to some edge in X. The edge domination number γ' (G) of G is the minimum cardinality taken over all edge dominating sets of G. An edge dominating set X is called an independent edge dominating set if no two edges in X are adjacent. The independent edge domination number γ'_i (G) of G is the minimum cardinality taken over all independent edge dominating sets of G. The edge independence number β_1 (G) is defined to be the number of edges in a maximum independent set of edges. Arumugam and S. Velammal [3] derived many results related to edge domination.

Labeling of a graph plays a vital role in graph theory. Seod M.A. and Youssef M.Z [4] and Joseph A. Gallian [5] dealt with graph labeling besides many others. A labeling of a graph G is an assignment of distinct positive integers to its vertices. A graph is a prime square dominating graph, if the vertices of graph G are labeled with positive integers such that the vertex labeled with composite number c is adjacent to the vertex named with prime number p if and only if p^2/c (p2 divides c).

II. IMPORTANT RESULTS

The following results obtained by S. R. Jayaram [6] characterize edge dominating sets of graphs and their edge domination number.

Result 1: For any graph G, $\gamma' = \gamma'_i$

Result 2: For any (p; q) -graph G; $\gamma' \leq q - \Delta'$ where Δ' denotes the maximum degree of an edge in G.

Result 3: For any (p; q) - graph G; $\gamma' = q - \beta_1 + q_0$ where q_0 is the number of isolated edges in G.
Result 4: For any (p; q) - graph G; $\gamma' + d' \leq q + 1$.

III. MAIN THEOREMS

A. Theorem 1

Let the vertex set V = { $v_1, v_2, v_3,..., v_n$}, n ≥ 2 of a prime square dominating graph G be such that a vertex v_i, 1≤ i ≤ n is labeled with a prime number p_1 and the remaining all vertices are labeled with composite numbers $c_2, c_3,... , c_n$ so that p_1^2 divides c_k for k = 2, 3, 4, …, n. Then the edge domination number of the prime square dominating graph G is one.

Proof: Let the vertex set V={ $v_1, v_2, v_3,…, v_n$} of the prime square dominating graph G be labeled with composite numbers and prime numbers satisfying the conditions mentioned in the theorem. Obviously, $\{p_1, c_k\}$ is an edge of the graph $for\ k = 2, 3, ... , n$ since p_1^2 divides c_k for k = 2, 3, 4, …, n. As there is no other vertex labeled with prime except p_1, c_k cannot be adjacent to any other vertex, except p_1 for k = 2, 3, 4, ……, n. Therefore, the distinct edges of the graph are $\{p_1, c_2\}$, $\{p_1, c_3\}$, ……, $\{p_1, c_n\}$ and moreover, every edge dominates every other edge of the graph. The set of every edge is an edge dominating set. Edge domination number of a graph is always greater than or equal to one. Hence, edge domination number of the graph is one.

1) Experimental problem: Consider the prime square dominating graph G with vertex set V ={3, 9, 18, 45, 63, }. The graph satisfies all the conditions stated in the above theorem for $p_1 = 3$, $c_2 = 9$, $c_3 = 18$, $c_4 = 45$, $c_5 = 63$. Clearly, minimum edge dominating sets are $\{p_1, c_k\}$, where k = 2, 3, …, 5. The edge domination number of the graph is one. Prime square dominating graph in this case will be as shown in the Fig. 1.

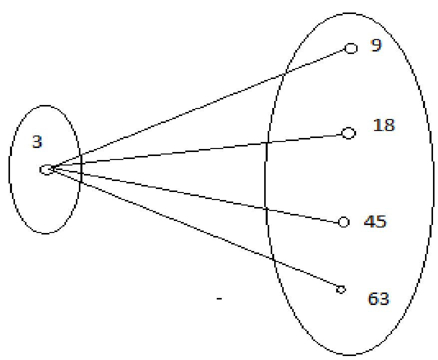

Fig. 1

B. Theorem 2

If the vertices v_i and v_j, where 1≤ i, j ≤ n and i≠j of the vertex set V = { $v_1, v_2, v_3,..., v_n$}, n ≥ 4 of a prime square dominating graph G are labeled with a prime number p_i and composite number c_j respectively and the remaining vertices are labeled with both composite and prime numbers so that p_i^2 divides all the composite numbers and all the squares of primes divide only c_j, then the edge domination number of the prime square dominating graph G is one.

Proof: Let the vertex set V = { $v_1, v_2, v_3, \ldots\ldots, v_n$}, of a prime square dominating graph G labeled with composite numbers and prime numbers satisfy the conditions mentioned in the theorem.

By hypothesis, except p_i, the squares of all the other primes divide only one composite number c_j. So, all the vertices labeled with primes, except p_i, are adjacent only to c_j. Moreover, all the composite numbers, except c_j, are divisible only by p_i^2. Vertex labeled with p_i is adjacent to all the vertices labeled with composite numbers and vertex labeled with c_j is adjacent to all the vertices labeled with prime numbers. It follows that, all the edges are adjacent to the edge $\{p_i, c_j\}$.The edge set consisting of single edge $\{p_i, c_j\}$ is an edge dominating set and further more it is a minimum edge dominating set. Hence, the edge domination number of the graph is one.

1) Experimental problem: Consider the prime square dominating graph with vertex set V = {2, 3, 5, 8, 900, 12, 16}. Prime square dominating graph in this case will be as shown in the Fig. 2.

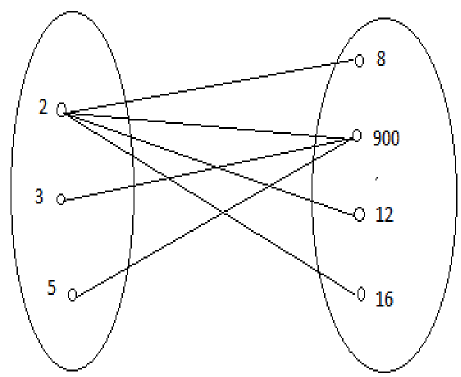

Fig. 2.

Here 2^2 divides 8, 900, 12, 16 and 3^2 divides only 900. Also, 5^2 divide only 900. The vertices p_i = 2 and c_j = 900 satisfy all the conditions mentioned in the theorem. The edge { 2, 900 } is adjacent to all the remaining edges. Hence, the set consisting of the single edge { 2, 900 } is a minimum edge dominating set. Hence, the edge domination number of the graph is one.

C. Theorem 3

Let G be a prime square dominating graph with vertex set V= { $v_1, v_2, v_3, \ldots, v_n$}, n ≥ 3. Let the vertices with even suffixes be labeled with prime numbers $p_2, p_4, p_6, \ldots$.respectively starting with $v_2$and the vertices with odd suffixes be labeled with composite numbers $c_1, c_3, c_5, \ldots$.respectively starting with v_1 such that p_i^2 divides only $c_{i-1}$and c_{i+1} for 1≤ i ≤ n and for even values of i. Then

$$\text{the edge domination number of the graph G} = \begin{cases} \frac{n}{3} & \text{for n = 3, 6, 9, …..} \\ \frac{n-1}{3} & \text{for n = 4, 7, 10, …..} \\ \frac{n+1}{3} & \text{for n = 5, 8, 11, …..} \end{cases}$$

Proof: Let the vertex set V = { $v_1, v_2, v_3, \ldots\ldots, v_n$} of the graph G be labeled with composite numbers and prime numbers satisfying the conditions mentioned in the theorem. It is supposed that p_i^2 divides only c_{i-1} and c_{i+1} for 1≤ i ≤ n and for even values of i. Two cases will arise i.e., n may be odd or even

Case (i): Let n be even. Then

p_2^2 divides only c_1 and c_3 ; p_4^2 divides only $c_3$and c_5 ; p_6^2 divides only $c_5$and c_7 ; …………….; p_n^2 divides only c_{n-1}. As the graph is a prime square dominating graph, the edges of the graph will be { c_1, p_2}, { p_2, c_3}, { c_3, p_4}, { p_4, c_5},……., { c_{n-3}, p_{n-2}}, { p_{n-2}, c_{n-1}},{ c_{n-1}, p_n} .

Case (ii): Let n be odd. Then

p_2^2 divides only c_1 and c_3; p_4^2 divides only c_3 and c_5; p_6^2 divides only c_5 and c_7; ……………….;

p_{n-1}^2 divides only c_{n-2} & c_n. The edges of the graph will be $\{c_1, p_2\}$,$\{p_2, c_3\}$, $\{c_3, p_4\}$, $\{p_4, c_5\}$, ……………… ….., $\{c_{n-4}, p_{n-3}\}$, $\{p_{n-3}, c_{n-2}\}$,$\{c_{n-2}, p_{n-1}\}$, $\{p_{n-1}, c_n\}$.

Clearly, the minimum edge dominating set of the graph G in the two possible cases will be as follows

$\{\{p_{n-1},c_n\},\{c_{n-4}, p_{n-3}\},\{p_{n-7},c_{n-6}\},\{c_{n-10}, p_{n-9}\},\ldots\ldots,\{p_2,c_3\}\}$ for n = 3, 9, 15, ….

$\{\{c_{n-1}, p_n\},\{p_{n-4},c_{n-3}\},\{c_{n-7}, p_{n-6}\},\{p_{n-10},c_{n-9}\},\ldots\ldots,\{p_2,c_3\}\}$ for n = 6, 12, 18, …..

$\{\{p_{n-2},c_{n-1}\},\{c_{n-5}, p_{n-4}\},\{p_{n-8},c_{n-7}\},\{c_{n-11}, p_{n-10}\},\ldots\ldots,\{p_2,c_3\}\}$ for n = 4, 10, 16, …..

$\{\{c_{n-2}, p_{n-1}\},\{p_{n-5},c_{n-4}\},\{c_{n-8}, p_{n-7}\},\{p_{n-11},c_{n-10}\},\},\ldots\ldots,\{p_2,c_3\}\}$ for n = 7, 13, 19, …..

$\{\{p_{n-1},c_n\},\{p_{n-3},c_{n-2}\},\{c_{n-6}, p_{n-5}\},\{p_{n-9},c_{n-8}\},\{c_{n-12}, p_{n-11}\},\ldots\ldots,\{p_2,c_3\}\}$ for n = 5, 11, 17, ….

$\{\{c_{n-1}, p_n\},\{c_{n-3}, p_{n-2}\},\{p_{n-6},c_{n-5}\},\{c_{n-9}, p_{n-8}\},\{p_{n-12},c_{n-11}\},\ldots\ldots,\{p_2,c_3\}\}$ for n = 8, 14, 20, …..

Hence, the edge domination number of the graph G =

$$\begin{cases} \frac{n}{3} & \text{for } n = 3, 6, 9, \ldots \\ \frac{n-1}{3} & \text{for } n = 4, 7, 10, \ldots \\ \frac{n+1}{3} & \text{for } n = 5, 8, 11, \ldots \end{cases}$$

1) Experimental problem: Consider the following prime square dominating graphs.

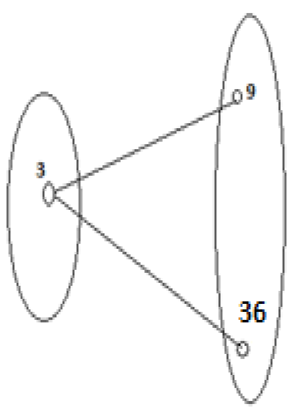

Fig. 3

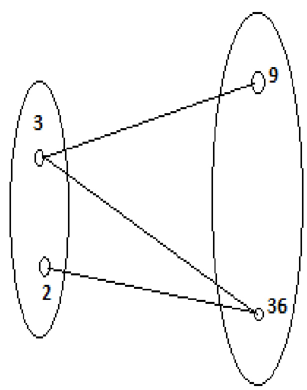

Fig.4

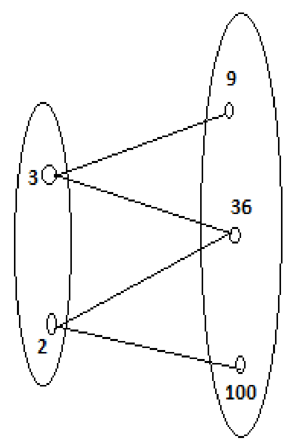

Fig.5

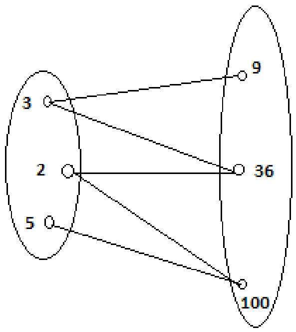

Fig.4

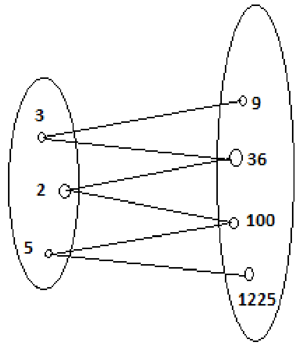

Fig.5

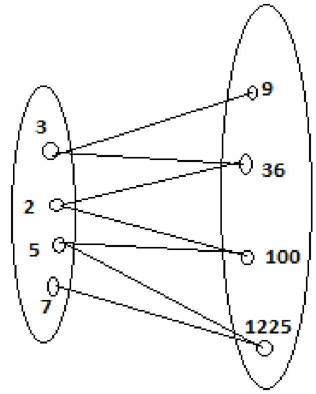

Fig.6

Prime Square Dominating Graphs

The above prime square dominating graphs satisfy all the conditions mentioned in the theorem for n=3, n=4, n=5, n=6, n=7 and n=8. Therefore, the minimum edge dominating sets of all the graphs in the above figures will be as in Table. I.

TABLE. I

Minimum Edge Dominating Sets of Prime Square Dominating Graphs

Fig.	No. of vertices ‘n’	Labeling of the vertices	Minimum edge dominating set	The edge domination number of the graph
Fig.3	3	$v_1 = c_1 = 9, v_2 = p_2 = 3, v_3 = c_3 = 36$	$\{\{p_2 = 3, c_3 = 36\}\}$	$\frac{n}{3} = \frac{3}{3} = 1$
Fig.4	4	$v_1 = c_1 = 9, v_2 = p_2 = 3, v_3 = c_3 = 36, v_4 = p_4 = 2$	$\{\{p_2 = 3, c_3 = 36\}\}$	$\frac{n-1}{3} = \frac{4-1}{3} = 1$
Fig.5	5	$v_1 = c_1 = 9, v_2 = p_2 = 3, v_3 = c_3 = 36, v_4 = p_4 = 2, v_5 = c_5 = 100$	$\{\{p_2 = 3, c_3 = 36\}, \{p_4 = 2, c_5 = 100\}\}$	$\frac{n+1}{3} = \frac{5+1}{3} = 2$
Fig.6	6	$v_1 = c_1 = 9, v_2 = p_2 = 3, v_3 = c_3 = 36, v_4 = p_4 = 2, v_5 = c_5 = 100, v_6 = p_6 = 5$	$\{\{p_2 = 3, c_3 = 36\}, \{c_5 = 100, p_6 = 5\}\}$	$\frac{n}{3} = \frac{6}{3} = 2$

Fig.7	7	$v_1 = c_1 = 9, v_2 = p_2 = 3,$ $v_3 = c_3 = 36, v_4 = p_4 = 2,$ $v_5 = c_5 = 100, v_6 = p_6 = 5$ $v_7 = c_7 = 1225$	$\{\{p_2 = 3, c_3 = 36\}, \{c_5 = 100, p_6 = 5\}\}$	$\frac{n-1}{3} = \frac{7-1}{3} = 2$
Fig.8	8	$v_1 = c_1 = 9, v_2 = p_2 = 3,$ $v_3 = c_3 = 36, v_4 = p_4 = 2$ $v_5 = c_5 = 100, v_6 = p_6 = 5$ $v_7 = c_7 = 1225, v_8 = p_8 = 7$	$\{\{p_2 = 3, c_3 = 36\}, \{c_5 = 100, p_6 = 5\}, \{c_7 = 1225, p_8 = 7\}\}$	$\frac{n+1}{3} = \frac{8+1}{3} = 3$

D. Theorem 4

Let G be a prime square dominating graph with vertex set V= $\{v_1, v_2, v_3, \ldots, v_{2n}\}$, n ≥ 4. Let the vertices with even suffixes be labeled with prime numbers $p_2, p_4, p_6, \ldots$respectively starting with v_2 and the vertices with odd suffixes be labeled with composite numbers $c_1, c_3, c_5, \ldots$respectively starting with v_1 such that p_i^2 divides only $c_{i-1}$and c_{i+1} for i = 2, 4, 6, ……2n-2 and p_{2n}^2 divides only $c_{2n-1}$and c_1. Then

$$\text{the edge domination number of the graph G} = \begin{cases} \frac{2n+2}{3} & \text{for } 2n = 4, 10, 16, \ldots \\ \frac{2n}{3} & \text{for } 2n = 6, 12, 18, \ldots \\ \frac{2n+1}{3} & \text{for } 2n = 8, 14, 20, \ldots \end{cases}$$

Proof: Let the vertex set V = $\{v_1, v_2, v_3, \ldots, v_{2n}\}$ of the graph G be labeled with composite numbers and prime numbers satisfying the conditions mentioned in the theorem. It is assumed that p_2^2 divides only c_1 *and* c_3 , p_4^2 divides only c_3 *and* c_5, p_6^2 divides c_5 and c_7, ……….. p_{2n-2}^2 divides c_{2n-3} and c_{2n-1} and p_{2n}^2 divides c_{2n-1} and c_1. As the graph is a prime square dominating graph, the distinct edges of the graph will be $\{c_1, p_2\}$, $\{p_2, c_3\}$, $\{c_3, p_4\}$, $\{p_4, c_5\}$, ……………………, $\{c_{2n-3}, p_{2n-2}\}$, $\{p_{2n-2}, c_{2n-1}\}$, $\{c_{2n-1}, p_{2n}\}$ and $\{p_{2n}, c_1\}$. For every three consecutive edges starting from the initial vertex c_1, every second edge can be placed in the edge dominating set. Every second edge in three consecutive edges is adjacent to the edges that precede it and follow it. As a result, the minimum edge dominating set of the graph G in all the possible cases will be as follows:

$\{\{p_{2n}, c_1\}, \{p_{2n-2}, c_{2n-1}\}, \{c_{2n-5}, p_{2n-4}\}, \{p_{2n-8}, c_{2n-7}\}, \{c_{2n-11}, p_{2n-10}\}, \ldots, \{p_2, c_3\}\}$ for 2n = 4, 10, 16, …..

$\{\{c_{2n-1}, p_{2n}\}, \{p_{2n-4}, c_{2n-3}\}, \{c_{2n-7}, p_{2n-6}\}, \{p_{2n-10}, c_{2n-9}\}, \ldots, \{p_2, c_3\}\}$ for 2n = 6, 12, 18, …..

$\{\{p_{2n}, c_1\}, \}, \{c_{2n-3}, p_{2n-2}\}\{p_{2n-6}, c_{2n-5}\}, \}, \{c_{2n-9}, p_{2n-8}\}\{p_{2n-12}, c_{2n-11}\}, \ldots, \{p_2, c_3\}\}$ for 2n = 8, 14, 20, …..

$$\text{Hence, the edge domination number of the graph G} = \begin{cases} \frac{2n+2}{3} & \text{for } 2n = 4, 10, 16, \ldots \\ \frac{2n}{3} & \text{for } 2n = 6, 12, 18, \ldots \\ \frac{2n+1}{3} & \text{for } 2n = 8, 14, 20, \ldots \end{cases}$$

1) Experimental problem: Consider the following prime square dominating graphs

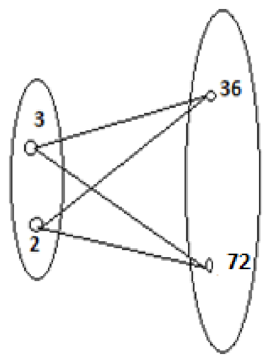

Fig. 9

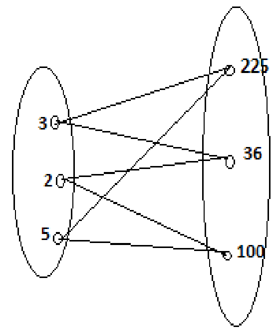

Fig. 10

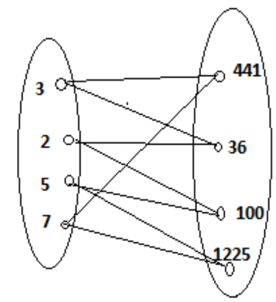

Fig. 11

Prime Square Dominating Graphs

The above prime square dominating graphs satisfy all the conditions mentioned in the theorem for n=4, n=6, n=7 and n=8. Therefore, the minimum edge dominating set of all the graphs in the above figures will be as in Table. II.

TABLE. II

Minimum Edge Dominating Sets of Some Prime Square Dominating Graphs

Fig.	No. of vertices '2n'	Labeling of the vertices	Minimum edge dominating set	The minimum edge domination number of the graph
Fig.9	4	$v_1 = c_1 = 36, v_2 = p_2 = 3,$ $v_3 = c_3 = 72, v_4 = p_4 = 2$	$\{\{c_1 = 36, p_4 = 2\}$ $\{p_2 = 3, c_3 = 72\}\}$	$\frac{2n+2}{3} = \frac{4+2}{3} = 2$
Fig.10	6	$v_1 = c_1 = 225, v_2 = p_2 = 3,$ $v_3 = c_3 = 36, v_4 = p_4 = 2$ $v_5 = c_5 = 100, v_6 = p_6 = 5$	$\{\{c_5 = 100, p_6 = 5\}$ $\{p_2 = 3, c_3 = 36\}\}$	$\frac{2n}{3} = \frac{6}{3} = 2$
Fig.11	8	$v_1 = c_1 = 441, v_2 = p_2 = 3,$ $v_3 = c_3 = 36, v_4 = p_4 = 2$ $v_5 = c_5 = 100, v_6 = p_6 = 5$ $v_7 = c_7 = 1225\ v_8 = p_8 = 7$	$\{\{p_8 = 7, c_1 = 441\},$ $\{c_5 = 100, p_6 = 5\},$ $\{p_2 = 3, c_3 = 36\}\}$	$\frac{2n+1}{3} = \frac{8+1}{3} = 3$

IV. CONCLUSION

Initially the paper has outlined the results regarding the edge domination in graphs and headed out into exploring the edge domination number of some prime square dominating graphs. This can be considered innovation in the field of theory related to prime square dominating graphs. In due course, efforts in the paper open up many an avenue in the field of research on prime square dominating graphs.

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