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RESEARCH ARTICLE

Automation of Mathematics Teaching — Demonstration by Methods of Completing the Square in Solving a Quadratic Equation

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***Abstract*—There were generally some steps used in traditional mathematics teaching, including writing down the origin of each topic or idea on the blackboard (maybe some topics lack of introducing the origins for simplicity), explanation of the theorems or properties, and proof, demonstration or practicing of some examples in solving steps, and corresponding homework assigned to students. Owing to the more progress in computer and network techniques than before, students learn by using these skills in many ways now, and get more help or assistance from the aided learning ways, for example, mathematics software wxMaxima helps to draw the graphs of functions and validate the computed results, and sometimes these modern technologies or facilities help students make up the absent classes by teaching files (audio-and-video or image files).**

This study plans to get a thorough and automatic way in learning mathematics by using the free and powerful software wxMaxima as a basis. The authors try to program the series of demonstration steps into codes in wxMaxima, and then integrate these steps into a final mathematical class for specific topic teaching use. First by use of a multimedia interface to introduce the source or origin of the topic to the beginner (maybe more interesting and attracting than the dull and pure texts statements), and then the full explanation of the theorems and inference of their properties (stress on the generality or universality on the theorems), examples and practicing and special instances occurring in this topic. Finally, assigning the homework for students to really interact with the software to solve their problems encountered in this learning stage, students can key in the same kind of problems to get the solving processes in a series of demonstration in wxMaxima and to contrast with the results of their self-practice.

***Keywords*—Symbolic algebraic system; Method of completing the square; Quadratic equation; Automatic demonstration**

I. A BRIEF GUIDE TO WXMAXIMA

The author thanks Professor Bill Schelter[1] and other contributors for their great efforts and dedication, and it can be said that people of the world are very fortunately to enjoy this sweet fruit. Professor Schelter was the midwife of this free software wxMaxima (formerly known as Macsyma system). He graduated from McGill University in Canada, and afterwards taught at the Department of Mathematics, University of Texas, US. He was good at the nature of artificial intelligence programming language LISP, and had excellent writing skills using it. In addition he had also a lot of special devotion to the implementation of GNU Common Lisp. Maxima was constantly improved from the development of a computer algebra system, Macsyma, which was developed at the Massachusetts Institute of Technology MIT in the 1960s commissioned by the U.S. Department of Energy DOE. As Professor Schelter strove and struggled for years, finally the US Department of Energy approved to

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release an open source program for subsequent users to update and make noncommercial use of it in 1998. Owing to the limit of article lengths, other information about how to install wxMaxima software and the detailed instructions using wxMaxima, can refer to webpages of Tsai [2], Wang [3], Hung [4], Chen [5], Shann [6] and others. In addition Maxima official website [7], Hand [8] or writings of Kuo and Chen [9-20] can also be as references.

II. INTRODUCTION

Currently the selling price of the commercial mathematical software such as Mathematica, Maple, Matlab, etc., even if the education version or deal price, is not cheap for students, and may be an economic burden especially for ones in developing countries or the Third World. Although the functions and interfaces of wxMaxima software cannot compare with those of commercial mathematical software, the advantage is the English instructions would not be too difficult to learn and remember, users can use common commands list to help learn the use of wxMaxima and, more importantly it is free and has pretty powerful computing capabilities.

Besides, the above mentioned computing software and wxMaxima all have the defect that answers were obtained almost immediately when the computing instructions were given, but unfortunately for learners with poor computing power, the answer can be quickly obtained, however, this answer how to be calculated still remained obscure in their mind, and it is a real regret for students or learners tardy in learning.

Turning the mathematical expressions or demonstration process step by step into an automatic screen display on computers is a new attempt and an automated teaching way based on familiarity with the use of wxMaxima and plentiful accumulation of experience in teaching mathematics, which would make the work of automation of mathematical deductive inference more perfect. Therefore if the automated demonstration of mathematical solving process and explanation can be finished, the author believes that when students have learning disabilities or doubts, they can contrast their practices with the demonstrated process by repetitive new solving examples showed by the automated demonstration program which could help instructors perform the remedial instruction more easily and effectively. Certainly the normal operation of computers on displaying the explanation process, demonstrating with logic and satisfying various special cases all need consideration in programming the demonstrated code in advance. For example, any system of linear equations of two unknowns may have three cases: inconsistent (no solution), independent (just only one solution), and dependent (infinitely many solutions). The programmed code must consider previously all occurring conditions and suitably shows its possibilities in the demonstrated stage of solving the system of linear equations. At this stage for the interpretation of the concept, it probably could not be demonstrated well and wholly by a computer display, the author hopes to have more progress in the ability of programming in artificial intelligent language LISP to improve the ways of expressing the more real explanation in classrooms, and then students could realize and understand the demonstration more clearly than before.

III. SIMPLE DESCRIPTION OF THEORY OF COMPLETING THE SQUARE IN SOLVING A QUADRATIC EQUATION

When a, b, and $c \in R$ (real numbers) , and $a \neq 0$, the expressions of this form $a x^2 + b x + c = 0$ are called quadratic equations (simply as quadratic), where a, b, and c are called coefficients of the quadratic, linear, and constant terms, respectively. Owing to a ($\neq 0$) , b , $c \in R$, they have many combinations, for example, like a=1 , b = 3 , c = 2 , a=2 , b = 4 , c = 2, and a=3 , b= 6 , c= 9 etc., and the corresponding quadratic equations are $x^2 + 3 x + 2 = 0$, $2 x^2 + 4 x + 2 = 0$, and $3 x^2 + 6 x + 9 = 0$ etc. Among so many choices of a, b, c, and related quadratic equation $a x^2 + b x + c = 0$, how to solve for the unknown variable x? And is there any solution existed? How about their solution types (one solution; no solution; infinitely many solutions)?

Theory	Explanation
$a x^2 + b x + c = 0$	If the coefficient of the quadratic term $a \neq 1$.
$a (x^2 + \frac{b}{a} x + \frac{c}{a}) = 0$	Then factor out a from each term in the left hand side of the equation.
$x^2 + \frac{b}{a} x + \frac{c}{a} = 0$	Divide a and obtain an effective equation

Note: Although the effective equation is different from the original equation in general, while their solutions are still the same, the proof is omitted for simplicity.

$x^2 + \frac{b}{a} x = -\frac{c}{a}$	Move the constant term $\frac{c}{a}$ to the right hand side of the equation

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$x^2 + \frac{b}{a}x + (\frac{b}{2a})^2 = -\frac{c}{a} + (\frac{b}{2a})^2$ Add the square of half of the coefficient of linear term $(\frac{b}{2a})^2$ to both sides of the equation.

$(x + \frac{b}{2a})^2 = -\frac{4ac}{4a^2} + \frac{b^2}{4a^2}$ The left hand side of the equation is a perfect square expression.

$(x + \frac{b}{2a})^2 = \frac{b^2-4ac}{4a^2}$ The value of a perfect square expression must be greater than or equal to zero.

When the unknown variable x is a real number, the perfect square $(x + \frac{b}{2a})^2$ is greater than or equal to zero, and so is the right hand side of the same equation, i.e., whether the numerator $(b^2 - 4ac)$ (called as the discriminant of the quadratic equation, and denoted by Δ) is greater than or equal to or even less than zero will be related to the existence of real roots of the unknown variable x.

1a) Discriminant $\Delta = b^2 - 4ac > 0$ (two distinct real roots)

From the expression $(x + \frac{b}{2a})^2 = \frac{b^2-4ac}{4a^2}$, it can be seen that $(x + \frac{b}{2a})$ is the square root of $\frac{b^2-4ac}{4a^2}$, i.e., $(x + \frac{b}{2a}) = \pm\sqrt{\frac{b^2-4ac}{4a^2}} = \pm\frac{\sqrt{b^2-4ac}}{2a}$, and it implies that $x = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a} = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$.

2a) Discriminant $\Delta = b^2 - 4ac = 0$ (a double root)

From the expression $(x + \frac{b}{2a})^2 = 0$, it is easy to see that $(x + \frac{b}{2a})$ is equal to zero and to solve for $x = -\frac{b}{2a}$.

3a) Discriminant $\Delta = b^2 - 4ac < 0$ (complex roots)

From the expression $(x + \frac{b}{2a})^2 = \frac{b^2-4ac}{4a^2}$, and if x is a real number, then no matter what real number x is, the expression $(x + \frac{b}{2a})$ is also a real value, and the square of it implies a positive value or zero, and the values of both sides of the equation are in different signs, and the real value solution of x does not exist, or the variable x does not have any real solution. When the considered number system is extended to complex system, the unknown variable x then have solution, i.e., $(x + \frac{b}{2a})^2 = \frac{b^2-4ac}{4a^2}$ is less than zero, $(x + \frac{b}{2a})$ is the complex root of $\frac{b^2-4ac}{4a^2}$, it implies that $(x + \frac{b}{2a}) = \pm\frac{\sqrt{b^2-4ac}}{2a} = \pm\frac{\sqrt{4ac-b^2}\,i}{2a}$, and finally $x = -\frac{b}{2a} \pm \frac{\sqrt{4ac-b^2}}{2a}i$ (complex roots).

Note: Imaginary unit $i = \sqrt{-1}$ (not in real number system) and if $r \in R^+$, it implies r is greater than 0, and complex number $\sqrt{-r} = \sqrt{-1 \cdot r} = \sqrt{-1} \cdot \sqrt{r} = i \cdot \sqrt{r} = \sqrt{r}\,i$.

The above mentioned method of solving the quadratic equation seems to stay in the early memory of teenage, although someone considered more about the solution and even some others did not take into consideration at all. However, how to make a systematic and not unidirectional introduction to the solution of the quadratic equations is very important to beginners.

Because there are infinitely many real examples of quadratic equation $a x^2 + b x + c = 0$ with which coefficients of a, b, and c associate. It is impossible to solve the corresponding equation example by example, while the description and explanation of the whole theory of solving the quadratic equation must emphasize that any combination of the coefficients of a, b, and c, i.e., no matter what real example of quadratic equation $a x^2 + b x + c = 0$ it is, belongs to one of the three cases: (1)discriminant $\Delta = b^2 - 4ac < 0$, (2)discriminant $\Delta = b^2 - 4ac = 0$, and (3) discriminant $\Delta = b^2 - 4ac > 0$ corresponding to (1) complex roots, (2) a double root , and (3) two distinct real roots, respectively. And the above statements should be the final induction of the whole theory of solving quadratic equations. For the real examples when discriminant $\Delta > 0$, Figures 2 and 3 show the case of quadratic equations with two distinct roots, Figures 4 and 5 for the real example of discriminant $\Delta = 0$ and $\Delta < 0$, respectively.

IV. REAL EXAMPLES OF AUTOMATIC DEMONSTRATION OF METHODS OF COMPLETING THE SQUARE IN SOLVING QUADRATIC EQUATIONS ON WXMAXIMA

By use of special functions of free mathematical software wxMaxima, the author writes the related demonstration program to show the process of solving quadratic equations automatically, the theory of completing the square in solving quadratic equations is stated as shown in Figure 1. And the real examples illustrating the three conditions of discriminant Δ of quadratic equations are shown in Figures 2~5. The symbols (%o1), (%o2), and …, (%on), …, etc., locating at the left side of Figures 1 to 5, denote the output sequence 1, 2, …, n, …, etc., and answer the input sequence 1, 2, …, n, …, etc. in wxMaxima. The percent symbol % is a

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specific control character in wxMaxima, o in (%on) is the first letter of the word "output", and n means the positive integer 1, 2, 3, …, etc.

```
(%o1) ------------------------------------------------------------------------
(%o2) The quadratic equations are expressions of the form a x^2 + b x + c = 0,
(%o3) where the coefficients a(≠0) , b , and c are any real numbers.
(%o4) ------------------------------------------------------------------------
(%o5) There are infinitely many choices of coefficients a(≠0), b, and c.
(%o6) So the quadratic equation a x^2 + b x + c = 0 have infinitely many forms.
(%o7) ------------------------------------------------------------------------
(%o8) If the coefficients a=3, b=2,and c=1, the equation is 3 x^2+ 2x + 1 = 0.
(%o9) ------------------------------------------------------------------------
(%o10) The question is how to solve the quadratic equation a x^2 + b x + c = 0.
(%o11) -----------------------------------------------------------------------
(%o12) The theory of methods of completing the square in solving the quadratic
(%o13) equation a x^2 + b x + c = 0 is stated as follows:
(%o14) -----------------------------------------------------------------------
(%o15) a x^2 + b x + c = 0
(%o16) -----------------------------------------------------------------------
(%o17) If the coefficient of the quadratic term a≠1, then factor out a from
(%o18) each term in the left hand side of the equation.
(%o19) If a=1, the action of factoring out a can be omitted.
(%o20) -----------------------------------------------------------------------
                                    b x   c
(%o21)                    a (x^2 + --- + -) = 0
                                    a   a
(%o22) -----------------------------------------------------------------------
(%o23) Divide a and obtain an effective equation (remaining the same solution).
(%o24) -----------------------------------------------------------------------
                                b x   c
(%o25)                     x^2 + --- + - = 0
                                a   a
(%o26) -----------------------------------------------------------------------
(%o27) Move the constant term to the right hand side of the equation
(%o28) and note for changing the sign
(%o29) -----------------------------------------------------------------------
                                b x     c
(%o30)                     x^2 + --- = - -
                                a     a
(%o31) -----------------------------------------------------------------------
(%o32) Add the square of half of the coefficient of linear term to
(%o33) both sides of the equation.
(%o34) -----------------------------------------------------------------------
                                b x   b^2     b^2   c
(%o35)                     x^2 + --- + --- = --- - -
                                a   4 a^2   4 a^2   a
(%o36) -----------------------------------------------------------------------
(%o37) The left hand side of the equation is a perfect square expression.
(%o38) -----------------------------------------------------------------------
                                  b       2   b^2 - 4 a c
(%o39)                       (x + ---)^2 = ---------
                                  2 a         4 a^2
(%o40) -----------------------------------------------------------------------
(%o41) The solution types of the equation (two distinct real roots,
(%o42) a double root or complex roots) depend on the values of
(%o43) the discriminant Δ = b^2 - 4 a c ( > or = or < 0 ).
(%o44) -----------------------------------------------------------------------
```

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```
(%o45) Define: Discriminant Δ = b^2 - 4 a c to classify the solution types
(%o46) of the quadratic equations into three groups (two distinct real
(%o47) roots, a double root or complex roots)
(%o48) ------------------------------------------------------------------------
(%o49)  (x + b/(2 a))^2 = (b^2 - 4 a c)/(4 a^2)
(%o50) ------------------------------------------------------------------------
(%o51)  ( x + b/2a ) has two possibilities :
(%o52) ------------------------------------------------------------------------
(%o53)  x + b/(2 a) = -sqrt(b^2 - 4 a c)/(2 a)
(%o54)  x + b/(2 a) = sqrt(b^2 - 4 a c)/(2 a)
(%o55) ------------------------------------------------------------------------
(%o56)  The solution of the quadratic equation is as follows :
(%o57) ------------------------------------------------------------------------
(%o58)  [x = (-sqrt(b^2 - 4 a c) + b)/(2 a), x = (sqrt(b^2 - 4 a c) - b)/(2 a)]
```

Fig. 1 The theory of completing the square in solving quadratic equations.

```
(%o1) ------------------------------------------------------------------------
(%o2) A real example of solving the quadratic equation a x^2 +
(%o3) b x + c = 0 by methods of completing the square
(%o4) ------------------------------------------------------------------------
(%o5) Case of two distinct real roots
(%o6) ------------------------------------------------------------------------
(%o7) First, enter the coefficient of quadratic term a (≠0)
(%o8) ------------------------------------------------------------------------
(%o10) a = 2
(%o11) -----------------------------------------------------------------------
(%o12) And then enter the coefficient of linear term b
(%o14) b = 6
(%o15) -----------------------------------------------------------------------
(%o16) Finally, enter the coefficient of constant term c
(%o17) -----------------------------------------------------------------------
(%o19) c = 4
(%o20) -----------------------------------------------------------------------
(%o21) The quadratic equation to be solved is as follows:
(%o22) -----------------------------------------------------------------------
(%o23) 2 x^2 + 6 x + 4 = 0
(%o24) -----------------------------------------------------------------------
(%o25) Determine the value of the discriminant Δ = b^2 - 4 a c
(%o26)  ( > , = , < ) 0 will affect the solution type of the equation.
(%o27) -----------------------------------------------------------------------
(%o28) The value of the discriminant Δ = b^2 - 4 a c is :
(%o29) -----------------------------------------------------------------------
(%o30) 4
(%o31) -----------------------------------------------------------------------
△ > 0 , the solution has two distinct real roots. =
△ > 0 , the solution has two distinct real roots.
(%o32) done
(%o33) -----------------------------------------------------------------------
(%o34) The steps shown below will illustrate the method of completing
(%o35) the square to solve the real equation of 2 x^2 + 6 x + 4 = 0.
(%o36) -----------------------------------------------------------------------
(%o37) 2 x^2 + 6 x + 4 = 0
```

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(%o38) ----------
(%o39) If the coefficient of the quadratic term a≠1, then factor out
(%o40) a from each term in the left hand side of the equation.
(%o41) If a=1, the action of factoring out a can be omitted.
(%o42) ----------
(%o43) $2(x^2+3x+2)=0$
(%o44) ----------
(%o45) Divide a and obtain an effective equation (the same solution).
(%o46) ----------
(%o47) $x^2+3x+2=0$
(%o48) ----------
(%o49) Move the constant term to the right hand side of the equation
(%o50) and remember to change the sign
(%o51) ----------
(%o52) $x^2+3x=-2$
(%o53) ----------
(%o54) Add the square of half of the coefficient of linear term to
(%o55) both sides of the equation.
(%o56) ----------
(%o57) $x^2+3x+\frac{9}{4}=\frac{1}{4}$
(%o58) ----------
(%o59) The left hand side of the equation is a perfect square expression.
(%o60) ----------
(%o61) $\left(x+\frac{3}{2}\right)^2=\frac{1}{4}$
(%o62) ----------
(%o63) (x + b/2a) has two possibilities :
(%o64) ----------
(%o65) $x+\frac{3}{2}=-\frac{1}{2}$
(%o66) $x+\frac{3}{2}=\frac{1}{2}$
(%o67) ----------
(%o68) The solution of the quadratic equation is as follows :
(%o69) ----------
(%o70) [x = - 2 , x = - 1]

Fig. 2 A real example of solving a quadratic equation (two distinct roots; a≠1).

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```
(%o1) ------------------------------------------------------------------------
(%o2) A real example of solving the quadratic equation a x^2 +
(%o3) b x + c = 0 by methods of completing the square
(%o4) ------------------------------------------------------------------------
(%o5) Case of two distinct real roots
(%o6) ------------------------------------------------------------------------
(%o7) First, enter the coefficient of quadratic term a (≠0)
(%o8) ------------------------------------------------------------------------
(%o10) a = 1
(%o11) -----------------------------------------------------------------------
(%o12) And then enter the coefficient of linear term b
(%o14) b = -5
(%o15) -----------------------------------------------------------------------
(%o16) Finally, enter the coefficient of constant term c
(%o17) -----------------------------------------------------------------------
(%o19) c = 6
(%o20) -----------------------------------------------------------------------
(%o21) The quadratic equation to be solved is as follows:
(%o22) -----------------------------------------------------------------------
(%o23) x^2 - 5 x + 6 = 0
(%o24) -----------------------------------------------------------------------
(%o25) Determine the value of the discriminant Δ = b^2 - 4 a c
(%o26)  ( > , = , < ) 0 will affect the solution type of the equation.
(%o27) -----------------------------------------------------------------------
(%o28) The value of the discriminant Δ = b^2 - 4 a c is :
(%o29) -----------------------------------------------------------------------
(%o30) 1
(%o31) -----------------------------------------------------------------------
△ > 0 , the solution has two distinct real roots.  =
△ > 0 , the solution has two distinct real roots.
(%o32) done
(%o33) -----------------------------------------------------------------------
(%o34) The steps shown below will illustrate the method of completing
(%o35) the square to solve the real equation of x^2 - 5 x + 6 = 0.
(%o36) -----------------------------------------------------------------------
(%o37) x^2 - 5 x + 6 = 0
(%o38) -----------------------------------------------------------------------
(%o39) If the coefficient of the quadratic term a≠1, then factor out
(%o40) a from each term in the left hand side of the equation.
(%o41) If a=1, the action of factoring out a can be omitted.
(%o42) -----------------------------------------------------------------------
(%o43) x^2 - 5 x + 6 = 0
(%o44) -----------------------------------------------------------------------
(%o45) Divide a and obtain an effective equation (the same solution).
(%o46) -----------------------------------------------------------------------
(%o47) x^2 - 5 x + 6 = 0
(%o48) -----------------------------------------------------------------------
(%o49) Move the constant term to the right hand side of the equation
(%o50) and remember to change the sign
(%o51) -----------------------------------------------------------------------
(%o52) x^2 - 5 x = - 6
(%o53) -----------------------------------------------------------------------
(%o54) Add the square of half of the coefficient of linear term to
(%o55) both sides of the equation.
```

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(%o56) ----------

(%o57) $x^2 - 5x + \frac{25}{4} = \frac{1}{4}$

(%o58) ----------

(%o59) The left hand side of the equation is a perfect square expression.

(%o60) ----------

(%o61) $\left(x - \frac{5}{2}\right)^2 = \frac{1}{4}$

(%o62) ----------

(%o63) (x + b/2a) has two possibilities :

(%o64) ----------

(%o65) $x - \frac{5}{2} = -\frac{1}{2}$

(%o66) $x - \frac{5}{2} = \frac{1}{2}$

(%o67) ----------

(%o68) The solution of the quadratic equation is as follows :

(%o69) ----------

(%o70) [x = 3 , x = 2]

Fig. 3 A real example of solving a quadratic equation (two distinct roots; a=1).

```
(%o1)  ------------------------------------------------------------------------
(%o2)  A real example of solving the quadratic equation a x^2 +
(%o3)  b x + c = 0 by methods of completing the square
(%o4)  ------------------------------------------------------------------------
(%o5)  Case of a double root
(%o6)  ------------------------------------------------------------------------
(%o7)  First, enter the coefficient of quadratic term a (≠0)
(%o8)  ------------------------------------------------------------------------
(%o10) a = 2
(%o11) ------------------------------------------------------------------------
(%o12) And then enter the coefficient of linear term b
(%o14) b = -12
(%o15) ------------------------------------------------------------------------
(%o16) Finally, enter the coefficient of constant term c
(%o17) ------------------------------------------------------------------------
(%o19) c = 18
(%o20) ------------------------------------------------------------------------
(%o21) The quadratic equation to be solved is as follows:
```

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```
(%o22) ------------------------------------------------------------------------
(%o23) 2 x^2 - 12 x + 18 = 0
(%o24) ------------------------------------------------------------------------
(%o25) Determine the value of the discriminant Δ = b^2 - 4 a c
(%o26)  ( > , = , < ) 0 will affect the solution type of the equation.
(%o27) ------------------------------------------------------------------------
(%o28) The value of the discriminant Δ = b^2 - 4 a c is :
(%o29) ------------------------------------------------------------------------
(%o30) 0
(%o31) ------------------------------------------------------------------------
△ = 0 , the solution has a double root. =
△ = 0 , the solution has a double root.
(%o32) done
(%o33) ------------------------------------------------------------------------
(%o34) The steps shown below will illustrate the method of completing
(%o35) the square to solve the real equation of 2 x^2 - 12 x + 18 = 0.
(%o36) ------------------------------------------------------------------------
(%o37) 2 x^2 - 12 x + 18 = 0
(%o38) ------------------------------------------------------------------------
(%o39) If the coefficient of the quadratic term a≠1, then factor out
(%o40) a from each term in the left hand side of the equation.
(%o41) If a=1, the action of factoring out a can be omitted.
(%o42) ------------------------------------------------------------------------
(%o43) 2(x^2 - 6 x + 9)= 0
(%o44) ------------------------------------------------------------------------
(%o45) Divide a and obtain an effective equation (the same solution).
(%o46) ------------------------------------------------------------------------
(%o47) x^2 - 6 x + 9 = 0
(%o48) ------------------------------------------------------------------------
(%o49) Move the constant term to the right hand side of the equation
(%o50) and remember to change the sign
(%o51) ------------------------------------------------------------------------
(%o52) x^2 - 6 x = - 9
(%o53) ------------------------------------------------------------------------
(%o54) Add the square of half of the coefficient of linear term to
(%o55) both sides of the equation.
(%o56) ------------------------------------------------------------------------
(%o57) x^2 - 6 x + 9 = 0
(%o58) ------------------------------------------------------------------------
(%o59) The left hand side of the equation is a perfect square expression.
(%o60) ------------------------------------------------------------------------
(%o61) (x - 3)^2 = 0
(%o62) ------------------------------------------------------------------------
(%o63)  ( x + b/2a ) has two possibilities :
(%o64) ------------------------------------------------------------------------
(%o65) x - 3 = 0
(%o66) x - 3 = 0
(%o67) ------------------------------------------------------------------------
(%o68)  The solution of the quadratic equation is as follows :
(%o69) ------------------------------------------------------------------------
(%o70) [ x = 3 ]
```

Fig. 4 A real example of solving a quadratic equation (a double root).

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```
(%o2) A real example of solving the quadratic equation a x^2 +
(%o3) b x + c = 0 by methods of completing the square
(%o4) -------------------------------------------------------------------------
(%o5) Case of complex roots
(%o6) -------------------------------------------------------------------------
(%o7) First, enter the coefficient of quadratic term a (≠0)
(%o8) -------------------------------------------------------------------------
(%o10) a = 2
(%o11) ------------------------------------------------------------------------
(%o12) And then enter the coefficient of linear term b
(%o14) b = -3
(%o15) ------------------------------------------------------------------------
(%o16) Finally, enter the coefficient of constant term c
(%o17) ------------------------------------------------------------------------
(%o19) c = 5
(%o20) ------------------------------------------------------------------------
(%o21) The quadratic equation to be solved is as follows:
(%o22) ------------------------------------------------------------------------
(%o23) 2 x^2 - 3 x + 5 = 0
(%o24) ------------------------------------------------------------------------
(%o25) Determine the value of the discriminant Δ = b^2 - 4 a c
(%o26)  ( > , = , < ) 0 will affect the solution type of the equation.
(%o27) ------------------------------------------------------------------------
(%o28) The value of the discriminant Δ = b^2 - 4 a c is :
(%o29) ------------------------------------------------------------------------
(%o30) -31
(%o31) ------------------------------------------------------------------------
△ < 0 , the solution has complex roots. =
△ < 0 , the solution has complex roots.
(%o32) done
(%o33) ------------------------------------------------------------------------
(%o34) The steps shown below will illustrate the method of completing
(%o35) the square to solve the real equation of 2 x^2 - 3 x + 5 = 0.
(%o36) ------------------------------------------------------------------------
(%o37) 2 x^2 - 3 x + 5 = 0
(%o38) ------------------------------------------------------------------------
(%o39) If the coefficient of the quadratic term a≠1, then factor out
(%o40) a from each term in the left hand side of the equation.
(%o41) If a=1, the action of factoring out a can be omitted.
(%o42) ------------------------------------------------------------------------
(%o43) 2 (x^2 - 3 x/2 + 5/2) = 0
(%o44) ------------------------------------------------------------------------
(%o45) Divide a and obtain an effective equation (the same solution).
(%o46) ------------------------------------------------------------------------
(%o47) x^2 - 3 x/2 + 5/2 = 0
(%o48) ------------------------------------------------------------------------
(%o49) Move the constant term to the right hand side of the equation
(%o50) and remember to change the sign
(%o51) ------------------------------------------------------------------------
(%o52) x^2 - 3 x/2 = -5/2
(%o53) ------------------------------------------------------------------------
(%o54) Add the square of half of the coefficient of linear term to
(%o55) both sides of the equation.
(%o56) ------------------------------------------------------------------------
```

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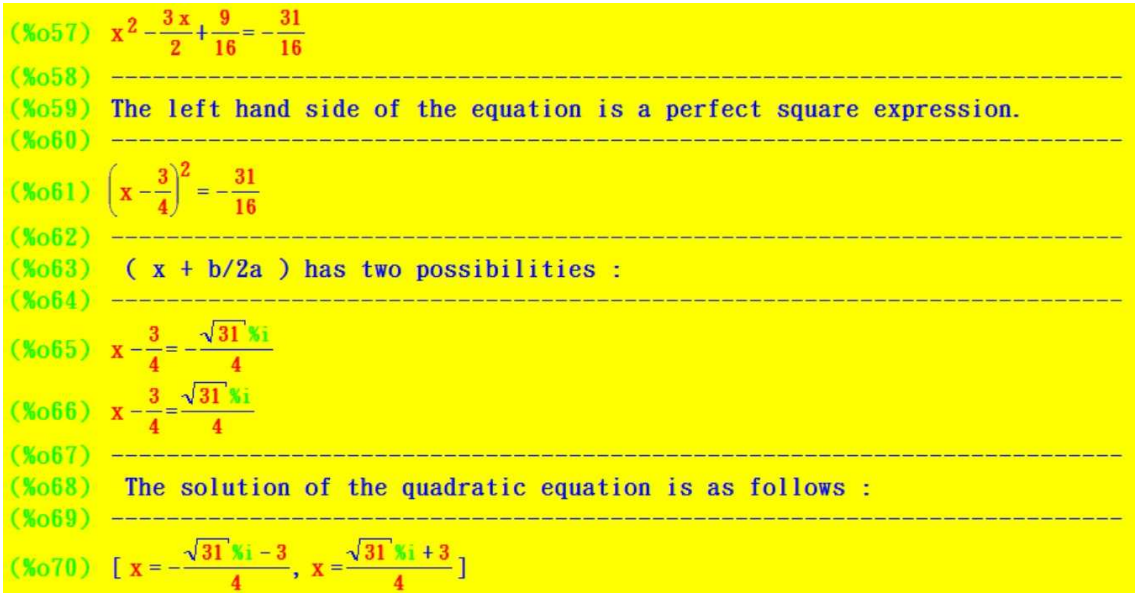

Fig. 5 A real example of solving the quadratic equation (complex root) and imaginary unit i is denoted by %i on wxMaxima.

V. CONCLUSIONS AND DISCUSSION

Most students couldn't realize well by only one lecture in classrooms, and the demonstration program in wxMaxima would help them to practice repeatedly by setting new examples of solving the quadratic equation for various choices of coefficients a, b, and c. They can examine the solving process of the real example of the chosen quadratic equation, and contrast their self-practice with the demonstrated processes shown by the program without limitation if the parameters or coefficients are set properly. Therefore students could find out the mistakes or error results which reflect their unfamiliar learning. By practicing many times in this way, students would understand the errors in their practice and amend the wrong concept. The author believes that it would help students enhance their confidence and outcomes of learning.

This study stresses that the work of statement and deduction of theory, or computing of real examples including tedious expressions could be process by wxMaxima, and the key point of learning must be transferred to the strengthening and establishment of conception, i.e., the important matter is to realize theory and conception clearly. The author reiterates that both of the understanding and familiarity of theory are important, while only paying attention to entering the coefficients of a, b, and c or familiarity with the operation on wxMaxima and finally forgetting the important conception are absolutely not the original intentions of the author. No matter what the statement of theory, or copious calculating process, students could hand them over to wxMaxima, and the author thinks that students would overcome their difficulty of learning full of confidence and be willing to face the future contents of learning.

This text takes the example of completing the square in solving the quadratic equation as an illustration on wxMaxima, if the coefficient of quadratic term a equals to 1, it is not necessary to factor out a form the quadratic equation, but it does not mean that any quadratic equation would conform to this kind of equation with a = 1 hereafter, and later the contents of learning would be so simple as this kind of problem of solving the quadratic equation. From a junior/senior high school to a college or a university, even the stage of an institute, there are different degrees of difficulty of learning, and more increasingly difficulty contents of learning would appear, for example, the calculation of Fourier series or Fourier transform, they are higher levels of mathematical computing, if students are unfamiliar with the calculation of differentials and integrals, it is impossible for them to learn well about the subsequent related mathematical computing. If students want to prevent learning obstacles or puzzle, they have to strengthen their mathematical skills in calculation to cope with computational problems from difficult concepts. Various engineering and scientific developments all base on the exploration of the nature of basic scientific principles. Inevitably, people have to rely on current developed mathematical model and to seek answers. Therefore different mathematical concepts such as Fourier integral, Fourier transform,…, etc., were derived from scientific principles, and a variety of corresponding applications are also developed such as the numeric analyses of audio, video or digital signals, from which the exploration of these concepts cannot separate.

The related computing in calculus or engineering mathematics has its theoretic statements and originals, and the deduction of symbolic algebra is inevitable in the process of theory. If instructors can provide automated demonstration of example calculations by programs on wxMaxima for students after class, the author believes

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that it would help students not fear the calculations in such course, and return to the recognition that good cognition and mastery of whole theory is the most important in learning. The ways of automated problem-solving program with the theoretical description of gradually showing solving steps on wxMaxima could assist students who are slow response or have poor computing capability, they can get the anthropomorphic explanation of solving exercises after school and a great help from this demonstration program on wxMaxima.

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