Hierarchal Polynomial Coding of Grayscale Lossless Image Compression

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Abstract: In this paper, a hierarchical scheme and linear polynomial coding is utilizing to compress image losslessly which is based on implementing the even/odd decomposition more than once, by exploring the mixing of image information, polynomial coefficients, and residual. The test results showed best performance of the hierarchal polynomial coding compared to the traditional polynomial coding.

Keywords: Hierarchal Scheme, Polynomial Coding, and Even/Odd Decomposition.

I. INTRODUCTION

Data compression systems have become an increasingly intensive and important research area over the last thirty years. Currently, data compression is a vast field encompassing many approaches and techniques [1]. Image compression techniques generally fall into two categories: lossless and lossy depending on the redundancy type exploited, where lossless is also called information preserving or error free techniques, in which the image is compressed without losing information as they rearrange or reorder the image content, they are based on the utilization of statistical redundancy alone (i.e., exploits coding redundancy and/or inter pixel redundancy) such as Huffman Coding, Arithmetic Coding and Lempel-Ziv Algorithm. While lossy removes content from the image, which degrades the compressed image quality, they are based on the utilization of psycho-visual redundancy, either solely or combined with statistical redundancy such as Vector quantizer, Fractal, and JPEG [2]. Reviews of lossless and lossy techniques can be found in [3-7].

Polynomial coding one of the modern image compression techniques remains in development, based on utilizing modelling concept of prediction and differentiation. The techniques characterized by its simplicity, symmetry of encoder and decoder and efficiency of use the coefficients and residual image [8], for more details see [9-12].

Generally, the hierarchal scheme representation can be divided into four general approaches, where the first methodology is based on multiresolution base that adopted by Burgett and Das (1991 and 1993) [13-14], utilized multi-resolution predictive coding which is an application of multi-layered images of wavelet decomposition and predictive coding technique. The results showed the efficiency in encoding time, and compression performance with high quality. Also Ghadah and Haider (2013) [15], applied a simple lossless techniques with the incorporation of multiresolution coding with the linear polynomial techniques, where the integrity of both techniques leads to high efficiency performance. The second methodology is based on...
utilization of **predictive coding once or multiple times** to remove the rest of the redundancy embedded between the estimated coefficients that developed by Das and Lin (1996) [16]. Several adaptations adopted by Ghadah (2014) [17] and Ghadah et al. (2017) [18] to improve the predictive coding performance with extended into three layers block base, whole image coefficients base, and correlated coefficients base only, respectively. The third methodology is based on exploited the interpolation base that used by Athraa (2015) [19], to constructed an adaptive polynomial compression system of nearest neighbour interpolation techniques through shrinking/enlarging base. The results showed an improvement of the adaptive techniques in terms of compression performance with high preserving the image quality. Lastly, the forth methodology is based on utilizing the separation base that differentiated the image into two sub images, namely an even sub image and an odd sub image using the row and/or column representation, this concept adopted by a number of refreshers such as [20-24], where the efficient performance indicated.  

This paper is concerned with improving the polynomial coding techniques using the hierarchal scheme to compress gray scale images losslessly efficiently, that presents the mixing of even and odd hierarchal decomposition effectively along the input image, the coefficients, the residual and the combination between them.  

The rest of paper organized as follows, section 2 contains comprehensive clarification of the proposed system; the results for the proposed system and the conclusions, is given in sections 3 and 4, respectively.

## II. THE PROPOSED COMPRESSION SYSTEM

The steps below illustrated the suggested system implementation in more details; Figure (1) shows the basic steps clearly:  

**Step 1:** Load the input uncompressed image $I$ of size $N 	imes N$ that corresponds to $layer_0$ or the root of the hierarchal representation.  

**Step 2:** Decompose $I$ image into even odd / sub-images ($I_e$ & $I_o$) corresponding to the first layer hierarchal scheme $layer_1$. Here the separation representation of even/odd or row/column base is adopted to remove the redundancy embedded within the input image, as illustrated in Figure (2).  

**Step 3:** Utilize the polynomial coding techniques to compress either even sub-image or odd sub-image representation corresponds to $layer_2$ such as:  

1. Partition the even sub-image $I_e$ into non-overlapping blocks of fixed sized $(n \times n)$ (i.e., 4x4, 8x8).  
2. Compute the coefficients of the linear approximation model, using the equations below [8]:

   \[
   a_0 = \frac{1}{n \times n} \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} I_e(i,j) \quad (1)
   \]

   \[
   a_i = \frac{\sum_{i=0}^{n-1} \sum_{j=0}^{n-1} I_e(i,j) \times (j-x_c)}{\sum_{i=0}^{n-1} \sum_{j=0}^{n-1} (j-x_c)^2} \quad (2)
   \]

   \[
   a_2 = \frac{\sum_{i=0}^{n-1} \sum_{j=0}^{n-1} I_e(i,j) \times (i-y_c)}{\sum_{i=0}^{n-1} \sum_{j=0}^{n-1} (i-y_c)^2} \quad (3)
   \]

   Where $I_e(i,j)$ is the even original sub image block of size $(n \times n)$ and

   \[
   xc = yc = \frac{n - 1}{2} \quad (4)
   \]

   Here the $(j-xc)$ and $(i-yc)$ corresponds to the variables of the polynomial that measure the distance of pixel coordinates to the block center ($xc$, $yc$). The $a_0$ coefficients represents the block mean, the $a_1$ and $a_2$ coefficients represents the ratio of sum pixel multiplied by the distance from the center to the squared distance in $i$ and $j$ coordinates respectively[8].

3. Create the predicted image value $\bar{I}_e$ using the computed polynomial coefficients for each encoded block representation:

   \[
   \bar{I}_e = a_0 + a_1(j-x_c) + a_2(i-yc) \quad (5)
   \]

4. Find the residual or prediction error as difference between the original $I_e$ and the predicted one $\bar{I}_e$.

   \[
   Res(i,j)=I_e(i,j) - \bar{I}_e(i,j) \quad (6)
   \]

**Step 4:** Extend the hierarchal scheme into two layers corresponding to $layer_1$ and $layer_2$ respectively, by exploring the residual image(s) of the preceding layer. In other words, third layer constructed by partition the residual image into even odd / sub-images ($Res_e$ & $Res_o$), then the even sub image exploited hierarchally to represents the fourth layer scheme ($Res_e$ & $Res_o$).

**Step 5:** Apply the symbol coding techniques to remove the coding redundancy between the values representation of the polynomial coefficients and the residual sub images.
**Fig. (1):** The hierarchal polynomial scheme of the proposed system.

**Fig. (2):** Input image that decomposed into even and odd sub-images [20].
III. EXPERIMENTAL RESULTS

For testing the proposed system performance; it is applied on three standard images, they are shown in figure 3 (a) ‘Lena’, 3 (b) ‘Rose’ and 3 (c) ‘Brain MRI’. Lena is characterized by a wide variety of image details, making it a complex highly detailed image, while Rose small or less variation in image detail, having large smooth areas with low detail. Brain lies somewhere between the two. All the images are a square of the same size, 256×256, and gray of 256 gray levels (8 bits/pixel).

![Figure 3: The three tested images of size 256×256, gray scale images, (a) Lena (b) Rose and (c) Brain.](image)

The compression ratio (CR), which is the ratio of the original image size to the compressed size, was adopted as a packing measure. Since, there is no degradation need to be evaluated in lossless compression where the decoded compressed image is identical to the original image, so the only guide here to the efficiency of proposed system is compression efficiency [15]. The results shown in tables (1-3) illustrate the comparison between the traditional polynomial coding and proposed hierarchal scheme of input image, polynomial coefficients, residual and the proposed system of extended layers using block sizes of 4×4. It is clear that the high performance compression ratio is attained of the proposed system due to utilization of effective hierarchal scheme of separation even/odd base compared to the traditional polynomial coding technique, due to removal of embedded redundancy efficiently.

### Table (1): Comparison performance between polynomial coding and proposed techniques of Hierarchal scheme for Lena image.

<table>
<thead>
<tr>
<th>Tested Image</th>
<th>Traditional Linear Polynomial Coding</th>
<th>Hierarchal scheme decomposition input image</th>
<th>Hierarchal scheme decomposition polynomial coefficients</th>
<th>Hierarchal scheme decomposition on residual</th>
<th>Proposed Hierarchal scheme decompositi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lena</td>
<td>CR</td>
<td>CR</td>
<td>CR</td>
<td>CR</td>
<td>CR</td>
</tr>
<tr>
<td></td>
<td>2.1489</td>
<td>3.8420/even</td>
<td>2.5984</td>
<td>4.3696</td>
<td>7.4950</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.8460/odd</td>
<td></td>
<td></td>
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</tbody>
</table>

### Table (2): Comparison performance between polynomial coding and proposed techniques of Hierarchal scheme for Rose image.

<table>
<thead>
<tr>
<th>Tested Image</th>
<th>Traditional Linear Polynomial Coding</th>
<th>Hierarchal scheme decomposition input image</th>
<th>Hierarchal scheme decomposition polynomial coefficients</th>
<th>Hierarchal scheme decomposition on residual</th>
<th>Proposed Hierarchal scheme decompositi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rose</td>
<td>CR</td>
<td>CR</td>
<td>CR</td>
<td>CR</td>
<td>CR</td>
</tr>
<tr>
<td></td>
<td>2.4419</td>
<td>4.3321/even</td>
<td>3.0490</td>
<td>4.4667</td>
<td>8.3337</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.3258/odd</td>
<td></td>
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</tbody>
</table>
Table (3): Comparison performance between polynomial coding and proposed techniques of Hierarchal scheme for Brain MRI image.

<table>
<thead>
<tr>
<th>Tested Image</th>
<th>Traditional Linear Polynomial Coding</th>
<th>Hierarchal scheme decomposition input image</th>
<th>Hierarchal scheme decomposition polynomial coefficients</th>
<th>Hierarchal scheme decomposition on residual</th>
<th>Proposed Hierarchal scheme decomposition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brain</td>
<td>CR 2.4206</td>
<td>CR 4.4504/even</td>
<td>CR 2.9176</td>
<td>CR 5.6351</td>
<td>CR 10.2689</td>
</tr>
<tr>
<td></td>
<td>4.4655/odd</td>
<td></td>
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</tbody>
</table>

Figures (4-6) summarizes the performance of the traditional polynomial coding and the proposed hierarchal scheme of the tested images.

Fig. (4): Comparison between the linear polynomial coding and the proposed techniques for Lena tested image.

Fig. (5): Comparison between the linear polynomial coding and the proposed techniques for Rose tested image.
Obviously, the compression ratio is directly affected by the image's characteristics or details, where for highly detailed images less compression ratio archived compared to images with small and moderate details.

IV. CONCLUSIONS
The proposed hierarchical scheme exploits the even/odd separation representation of the mixed between the image, coefficients and residual to improve the performance results compared to traditional polynomial coding technique of linear base is characterized by the simplicity, with low compression results that directly affected by the symbol encoder techniques. The results clearly shows the better hierarchal performance than the traditional polynomial coding, more compression ratio achieved (i.e., four times on average) with identical image quality.

REFERENCES