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RESEARCH ARTICLE

Time-Frequency Domain Characterization of Stationary and Non stationary Signals

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Abstract-- This paper presents the various methods for the spectral analysis of signals for the stationary as well as non-stationary signals. Due to non-stationary characteristics of the signals, it has been always a challenge to achieve time frequency distribution of such signals. Between the various techniques of signal analysis, this paper uses Fourier transform, Short time Fourier transform, wavelet transform, and Hilbert Huang transform for the analysis of stationary as well as non-stationary signals. A comparison between these frequency transformation techniques has been made by analyzing four types of test signals. The result shows the best method for the analysis of each type of test signal.

Keywords---- DFT, STFT, Time frequency transformations, HHT, HMS

I. INTRODUCTION

The signals are generated by the systems and they contain the information about the systems from where they are originated. To extract information from signals and reveal the underlying dynamics that corresponds to the signals, proper signal processing technique is needed. Typically, the process of signal processing transforms a time-domain signal into another domain, with the purpose of extracting the characteristic information embedded within the time series that is otherwise not readily observable in its original form. So the spectral analysis techniques are used to extract the maximum possible information from the signal so that maximum analysis of the system can be made in both time as well as frequency domain.

Depending upon the analysis techniques, the signals can be broadly classified under four categories. These four types are based on whether the signal is stationary or non-stationary, whether it contains single frequency component or multiple frequency components. The four signal types are listed in Table I along with their properties in the time domain.

This paper explains the impact of classical Heisenberg's uncertainty principle [4], which states that the product of temporal and frequency resolution is constant, over the various signal analysis techniques for each type of signal as given in Table I. This paper gives the suitable technique for each type of signal that gives the best results in terms of signal analysis.

TABLE 1: Types of signals and their time domain properties

Type	Time Domain properties
Type I	Stationary with single frequency component
Type II	Stationary with multiple frequency components
Type III	Non-stationary with single frequency component at a time
Type IV	Non-stationary with multiple frequency components

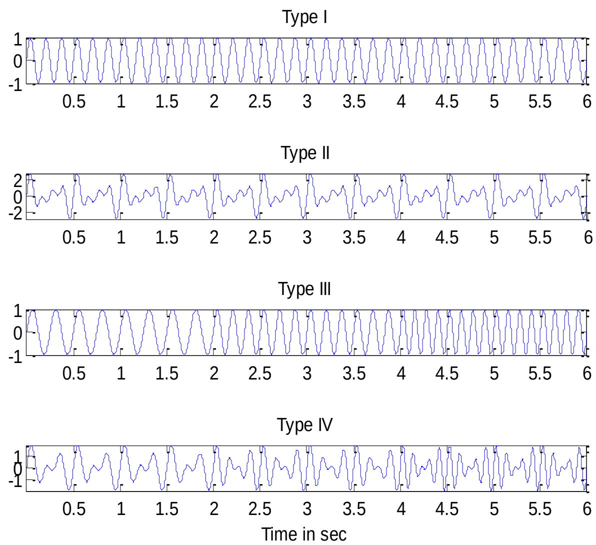

Fig. 1: Four types of signal as elaborated in Table 1

The signals Type I-IV are created as per the following equations

$$X_I(t) = \sin(2\pi 6t) \quad \text{for } 0 < t < 6$$

$$X_{II}(t) = \sin(2\pi 4t) + \sin(2\pi 6t) + \sin(2\pi 8t) \quad \text{for } 0 < t < 6$$

$$X_{III}(t) = \sin(2\pi 4t_1) + \sin(2\pi 6t_2) + \sin(2\pi 8t_3) \quad \text{for } \begin{matrix} 0 < t_1 < 2 \\ 2 < t_2 < 4 \\ 4 < t_3 < 6 \end{matrix}$$

$$X_{IV}(t) = \sin(2\pi 4t_1) + \sin(2\pi 6t_1) + \sin(2\pi 6t_2) + \sin(2\pi 8t_2) + \sin(2\pi 8t_3) + \sin(2\pi 10t_3) \quad \text{for } \begin{matrix} 0 < t_1 < 2 \\ 2 < t_2 < 4 \\ 4 < t_3 < 6 \end{matrix}$$

As stated in table I, the type I signal is a stationary signal containing frequencies of 6 Hz throughout the signal duration. The type II signal is also a stationary signal that contains frequencies of 4, 6 and 8Hz throughout the signal. The type III signal is a non-stationary signal containing 4 Hz for first 2 seconds, 6Hz for next 2 and 8Hz for last 2 seconds. Last signal i.e Type IV is a non-stationary signal containing 4 and 6 Hz, 6 and 8 Hz, and & 8Hz and 10 Hz for two seconds duration each

II. FOURIER TRANSFORM

The Fourier transform is probably the most widely applied signal processing tool in science and engineering. It reveals the frequency composition of a time series x(t), by transforming it from the time domain into the frequency domain. In 1807, the French mathematician Joseph Fourier found that any periodic signal can be presented by a weighted sum of a series of sine and cosine functions [8].
The Fourier transform of a signal x(t) can be expressed as

$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-2\pi ft}dt$$

The inverse Fourier transform that transforms signal back to its original domain, is given by [1]

$$x(t) = \int_{-\infty}^{\infty} X(f)e^{2\pi ft}df$$

The signals which are obtained from data acquisition system experimentally are generally sampled at discrete time intervals Δt, instead of continuously, within a total measurement time T. Such a signal x(n), can be transformed into frequency domain using Discrete Fourier Transform (DFT), defined as [4],

$$X(k) = \sum_{k=0}^{N-1} x(n)e^{-j2\pi nk/N}$$

Where N is the period of the signal, n is the number of sample.
The inverse Discrete Fourier Transform can be obtained by

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)e^{j2\pi nk/N}$$

Fourier transform is actually the convolution of signal x(t) or x(n) with sine and cosine function as understood from above equations. This convolution actually measures the similarity between signal x(t) and corresponding sine and cosine terms and then calculates the average over entire time range. This process is repeated after changing the frequency of sine and cosine terms. This can be graphically illustrated from the Fig. 2.

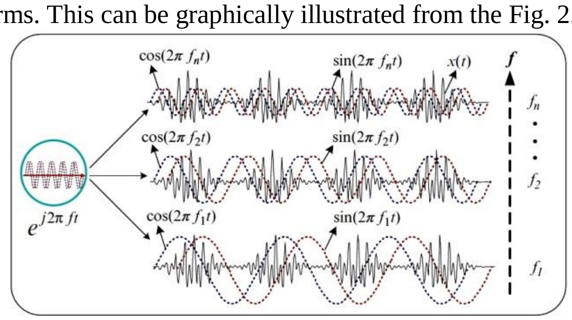

Fig. 2: Illustration of Fourier transform

While calculation of the Fourier transform or Discrete Fourier transform, the phenomenon of leakage and aliasing may play its role. Leakage occurs, when signal is extended periodically for performing DFT. This can be solved by using windows in DFT, however use of window may contribute towards frequency information of the signal. The aliasing occurs when the Shannon's sampling theorem is not satisfied. This causes the original frequency component to appear at the incorrect location. This can be easily solved by keeping the sampling frequency more than twice of highest frequency present in the signal.

The above said statements can be verified by applying the Fourier transform on the four types of signal as given in Table 1

The Fig. 3 shows the Fourier transform of the signals Type I-IV. From the figure it is quite clear the Fourier transforms fails to provide the precise value of the frequency present in the signal. This is due to the leakage effect as explained earlier. Moreover, Fourier transform also do not reveals how the frequency components vary with time. It is evident from the plot of Type II and Type III which appear to be similar in frequency domain; however they are quite distinct in time domain. So here we can conclude that the Fourier transform fails to provide frequency information with respect to time variation. This is considered a major drawback of Fourier transform and hence it is not suitable for non-stationary signals which vary with time

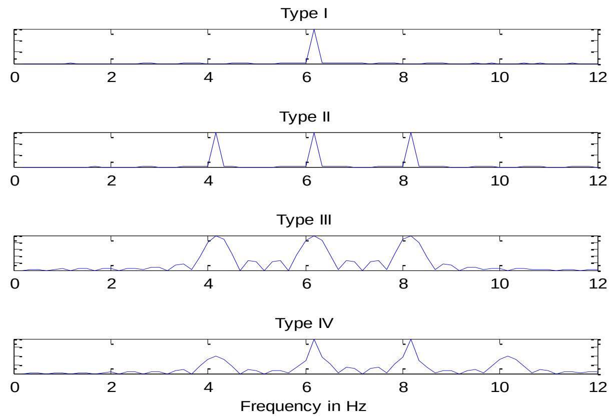

Fig. 3: Fourier Transform of signals as elaborated in Table 1.

III.SHORT TIME FREQUENCY TRANSFORM

The most nearest solution to above stated problem is the Short time Frequency transform. It utilizes the concept of the window of certain length that glides through the time axis and helps to calculate the Time Localized Fourier transform [3]. This concept was first introduced by Dennis Gabor in his paper titled "Theory of communication," published in 1946 [3].

The Short Time Fourier Transform (STFT) is given by

$$\mathrm{STFT}(\tau, f) = \int x(t)g(t-\tau)e^{-2\pi ft}dt$$

The STFT employs a time window g(t) of certain duration that is centered at time τ. For each specific time τ, the time localized Fourier transform is calculated. Then this window moves along the time axis and again the Fourier transform is calculated. This process is continued till complete signal is analyzed. This can be easily understood from the Fig. 4.

The most important and crucial factor for calculation of STFT is the duration of the window used. According to Heisenberg Uncertainty principle the precise knowledge of frequency present at the particular instant cannot be known, so this leads to the fact that reducing of the window size results in poor frequency resolution and good frequency resolution requires wide time window.

The above said fact is evident when the STFT is employed over signals Type II-IV with different window durations.

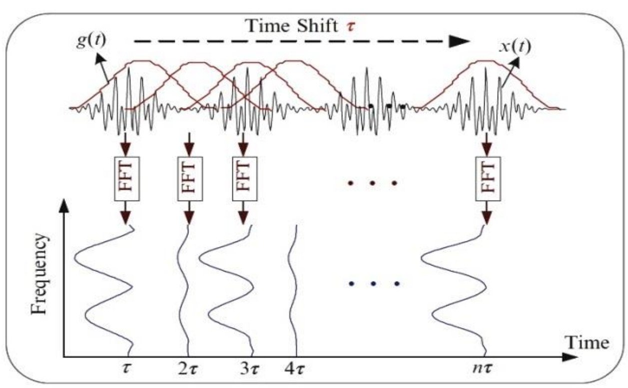

Fig. 4: Illustration of Short Time Fourier Transform

The STFT is applied at above said signals with three window lengths of 500ms, 700ms and 1 sec.

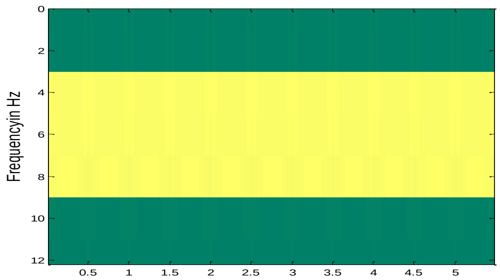

Fig. 5: STFT of Type II signal with window of 500ms

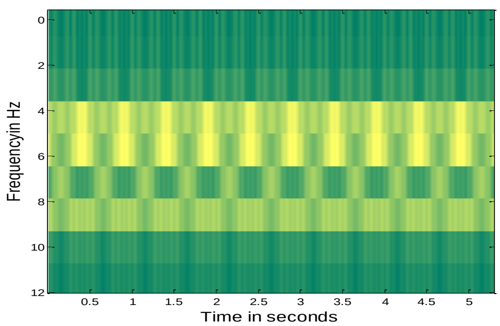

Fig. 6: STFT of Type II with window of 700 ms

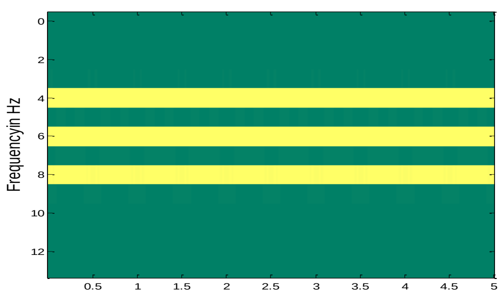

Fig. 7: STFT of Type II with window of 1 Sec

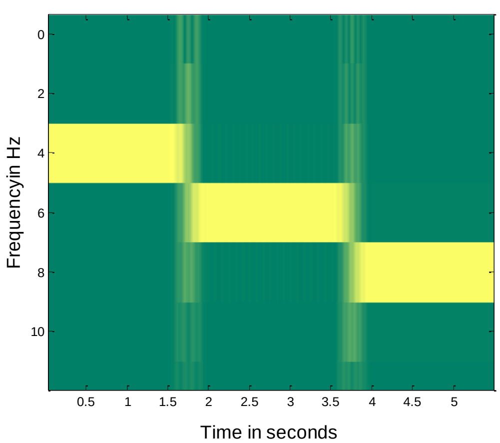

Fig. 8: STFT of Type III with window of 500 ms

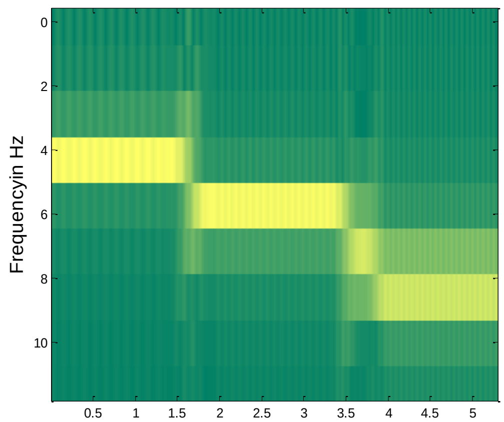

Fig. 9: STFT of Type III with window of 700 ms

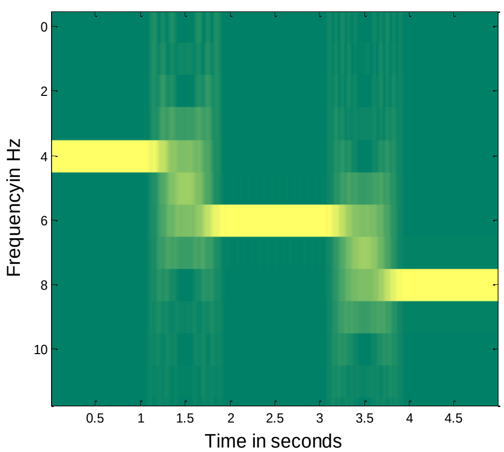

Fig. 10: STFT of Type III with window of 1 sec

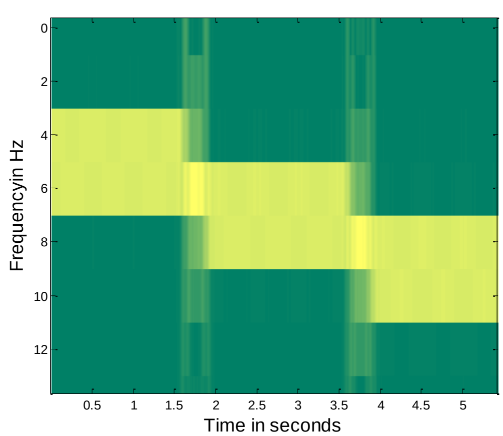

Fig. 11: STFT of Type IV with window of 500 ms

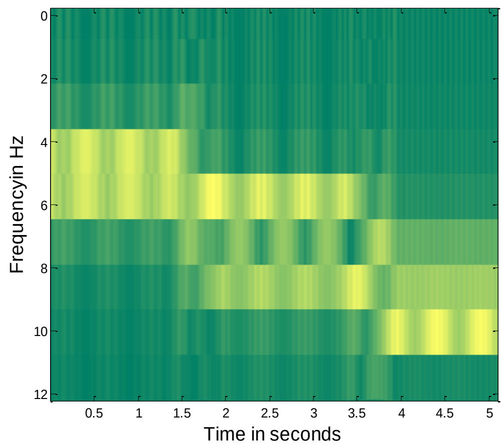

Fig. 12: STFT of Type IV of window of 700 ms

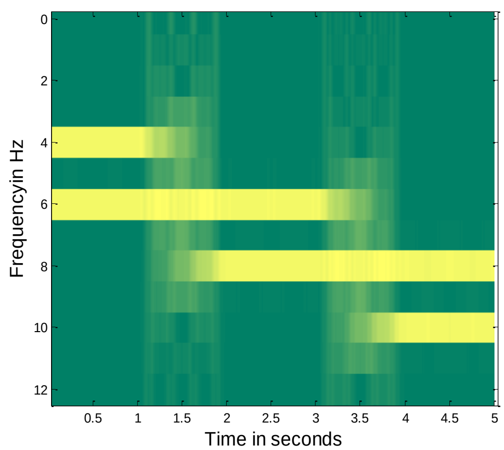

Fig. 13: STFT of Type IV with window of 1 sec

From the Fig 5-7, it is quite clear that STFT with small window duration is not capable of separating the close frequencies. As we increased the window duration from 500 ms to 1 second, the STFT clearly separated the three different frequencies. As Type II is a stationary signal, so STFT with a large window of normal Fourier Transform is best suited for this type of signal.

From Fig. 8-10, the role of Heinsenberg uncertainty principle is quite evident. In Fig 8, when window was of 500 ms duration, the frequency resolution is quite poor but, these frequencies are much clearly separated in time. As soon as, we increase the duration of window to 700ms, see Fig 9, the frequency resolution improved but, the time resolution degraded. Further increase in window size to 1 sec, Fig 10, improved frequency resolution more but again it degraded the time resolution. Similar results were seen in Fig 11-13, with Type IV signal, as we applied STFT with 500ms, 700 ms and 1 second window duration.

So here we can conclude that, the window size is crucial deciding factor in evaluation of STFT. It needs to be small if, frequencies separation is large, but time separation is small. Similarly, window size needs to be small, if frequencies separation is large, but time separation is small.

Now as in STFT, the window size is fixed, so it works against it, and appears to be its drawback. The inherent drawback of the STFT motivates researchers to look for other techniques that are better suited for processing non-stationary signals.

IV. WAVELET TRANSFORM

In contrast to the STFT technique where the window size is fixed, the wavelet transform enables variable window sizes in analyzing different frequency components within a signal[10] This is realized by comparing the signal with a set of template functions obtained from the scaling (i.e., dilation and contraction) and shift (i.e., translation along the time axis) of a base wavelet ψ(t) and looking for their similarities, as illustrated in following equation.

$$wt(s,t) = \frac{1}{\sqrt{s}}\int_{-\infty}^{\infty} x(t)\psi(\frac{t-\tau}{s})dt$$

Where the symbol s > 0 represents the scaling parameter, which determines the time and frequency resolutions of the scaled base wavelet ψ(t-τ)/s. The specific values of s are inversely proportional to the frequency[12]. The symbol t is the shifting parameter, which translates the scaled wavelet along the time axis. The symbol ψ*( ) denotes the complex conjugation of the base wavelet ψ(t). The wavelet transform is best illustrated by the following Fig. 14

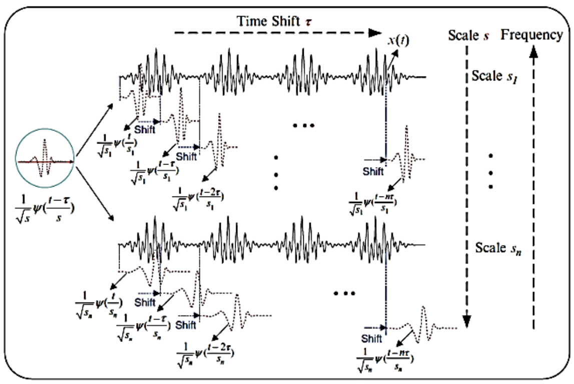

Fig. 14: Illustration of Wavelet Transform

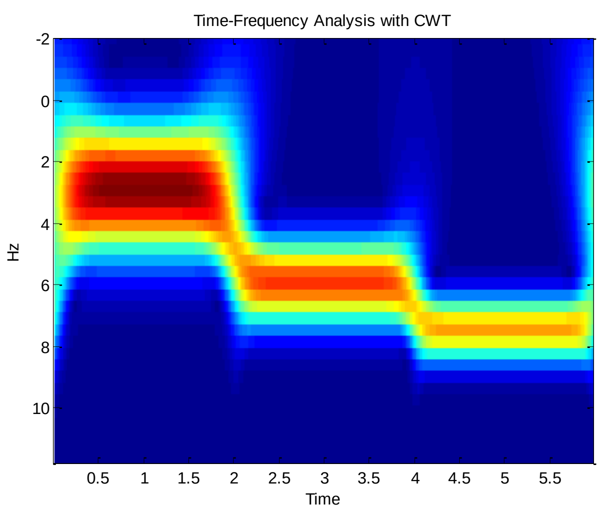

Fig. 15: Wavelet transform of Type III

As it can be seen from the figure that wavelet transform utilizes multi-resolution technique so it uses small window for high frequencies and large window for low frequencies. It is the scaling factor that plays the role in Multi-resolution analysis.[18-22]

For use of wavelet transform, the choice of mother wavelet is very important. Incorrect choice may result in wrong decomposition of the signal [14]. So there are different mother wavelets for different type of signals [15]

V. HILBERT HUANG TRANSFORM (HHT)

Hilbert Huang Transform (HHT) is named after David Hilbert. These are the statistical tools that use independent component analysis.

The Hilbert-Huang transform utilizes empirical mode decomposition (EMD) for the signal analysis. HHT is the emerging novel technique of signal decomposition having many interesting properties [23]. In particular, HHT has been applied to numerous scientific investigations, such as biomedical signals processing, geophysics, image processing, structural testing, fault diagnosis, nuclear physics and so on [24-27]. In order to facilitate the reading of this paper we will introduce in detail the Hilbert-Huang transformation, which is a relatively novel technique.

HHT uses EMD to decompose the signal into various intrinsic mode functions (IMF) and then Hilbert-Huang transform is applied to each IMF, therefore time frequency distribution is obtained.

Hilbert Huang transform gives the instantaneous frequency by differentiating the instantaneous angle with respect to time. The key to this is that signal should contain only single frequency component at a time so it require EMD to decompose the signal, in order to get correct HHT.

The empirical mode decomposition (EMD) method is developed from the simple assumption that any signal consists of different simple intrinsic mode oscillations.

The essence of the method is to identify the intrinsic mode functions (IMFs) by their characteristic time scale in the signal and then decompose the signal accordingly [25]. The characteristics time scale is defined by the time lapse between the successive extremes.

To extract the IMF from a given data set, a sifting process is implemented as follows. First, identify all the local extrema, and then connect all of the local maxima by a cubic spline line as the upper envelope.

Then, repeat the procedure for the local minima to produce the lower envelope [26]. The upper and lower envelopes should cover all the data between them.

Their mean is designated $m_1(t)$, and the difference between the data and $m_1(t)$ is $h_1(t)$, i.e.,

$$x(t)-m_1(t)=h_1(t)$$

To check if $h_1(t)$ is an IMF, we demand the following conditions:

- $h_1(t)$ should be free of riding waves, i.e., the first component should not display under-shots or over-shots riding on the data and producing local extremes without zero crossings.
- To display symmetry of the upper and lower envelope with respect to zero.
- Obviously the number of zero crossing and extremes should be the same in both functions.

The sifting process has to be repeated as many times as it is required to reduce the extracted signal to an IMF. In the subsequent sifting process steps, $h_1(t)$ is treated as the data. Then

$$h_1(t) – m_{11}(t) = h_{11}(t)$$

Where $m_{11}(t)$ is the mean of the upper and lower envelopes $h_1(t)$. This process can be repeated up to k times; $h_{1k}(t)$ is then given by [26]

$$h_{1(k-1)(}t) – m_{1k}(t) = h_{1k}(t)$$

Having obtained the IMFs by using EMD method, one applies the Hilbert transform to each IMF component [26]

$$H[h_{1k}(t)] = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{h_{1k}(t)}{t-\tau}d\tau$$

The Hilbert transform is then used to achieve the analytical signal which is given by [26]

$$z_i(t) = h_{1k}(t) + j\,H[h_{1k}(t)]$$

The analytical signal can alos be expressed as

$$z_i(t) = a_i(t)\exp(j\omega_i(t))$$

With amplitudes ai(t) and phase θi(t) defined by

$$a_i(t) = \sqrt{{h_{1k}}^2(t) + H^2[h_{1k}(t)]}$$

$$\theta_i(t) = \arctan(\frac{H[h_{1k}(t)]}{h_{1k}(t)})$$

The instantaneous frequency can be calculated as

$$\omega_i(t) = \frac{d\theta_i(t)}{dt}$$

This helps us to represent the amplitude and the instantaneous frequency in a three-dimensional plot, in which the amplitude is the height in the time frequency plane. This time-frequency distribution is designated as the Hilbert-Huang spectrum.[26]

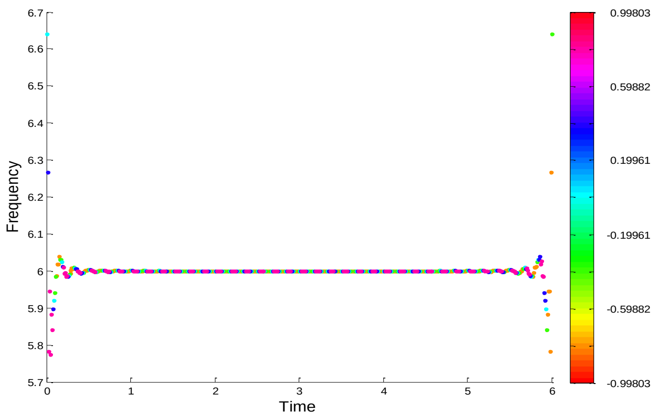

Fig. 16: HHT of Type I signal

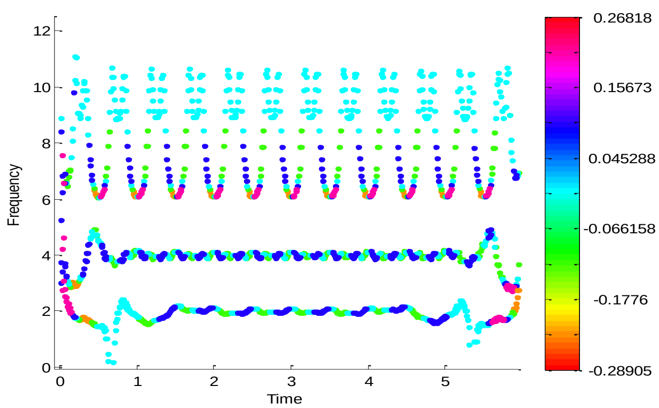

Fig. 17: HHT of Type II signal

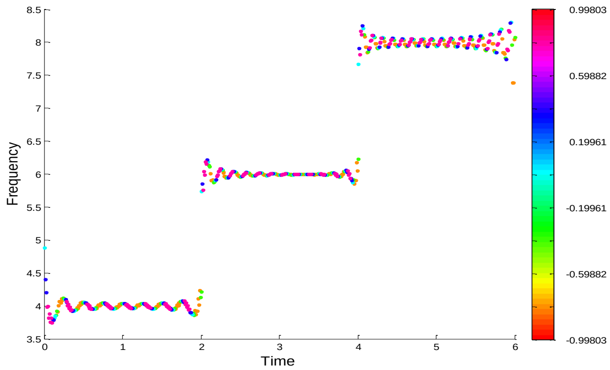

Fig. 18: HHT of Type III signal

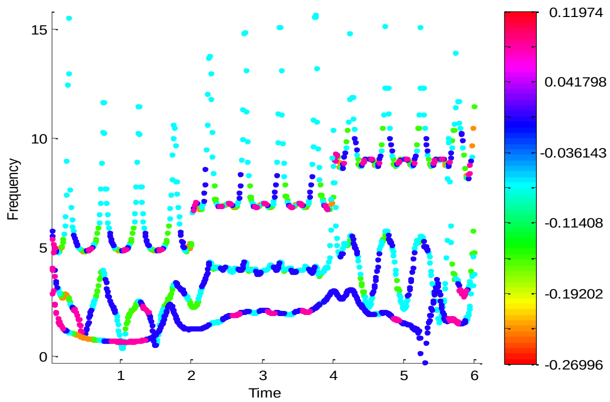

Fig. 19: HHT of Type IV signal

HHT of various signals has been given from the Fig. 16-19. From the Fig. 16 and 17, it can be inferred that, the HHT transform works well for those signals that contains only single frequency at a time, no matter whether it is stationary or non-stationary. From the Fig 17 and 19, we can see some extra frequency harmonics. These are due to errors introduced while decomposition of the signals such as end effects. So it is clear that, HHT performs poor for those signals that contain multiple frequencies at a time. The comparative analysis of all the techniques is given in Table II

Type	Time Domain properties	Best suitable transformation technique
Type I	Stationary with single frequency component	Hilbert Huang transform, Fourier transform
Type II	Stationary with multiple frequency components	Wavelet transform
Type III	Non-stationary with single frequency component at a time	Hilbert Huang transform
Type IV	Non-stationary with multiple frequency components	Wavelet transform

VI. CONCLUSION

This paper presents a comparative study of some Frequency transformation techniques. These time-frequency techniques are applied to four types of test signals and their results are compared.

It is concluded that for stationary signals Fourier transform allows us to clearly analyze the signal in its frequency transform without much variations. In case of Non-stationary signals, that do not have large frequency variations, STFT performs well, although there is always a compromise between frequency and time resolution. In case the non-stationary signals that have large frequency variations in term of sampling frequency, wavelet transform performs better than other techniques. The signals that contains single frequency component, the Hilbert Huang transform is the best approach, as it is independent component analysis technique and it shows clear cut frequency variation with time.

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