



# A Review of Rician Noise Reduction in MRI Images using Wave Atom Transform

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**Abstract :** *Magnetic resonance imaging is a medical imaging technique that measures the response of atomic nuclei of body tissues to high frequency radio waves when placed in a strong magnetic field and that produces images of the internal organs. De-noising is always a challenging problem in magnetic resonance imaging and important for clinical diagnosis and computerized analysis, such as tissue classification and segmentation. It is well known that the noise in magnetic resonance imaging has a Rician distribution. . In this paper, an improved de-noising technique is proposed on Magnetic Resonance Images highly corrupted with Rician Noise using wave atom shrinkage.*

## **General Terms**

*Clinical diagnosis, tissue classification, segmentation*

## **Keywords**

*De-noising, Histogram, Magnetic Resonance Image, Rician Noise, Variance Estimation, Wave Atom Transform*

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## I. INTRODUCTION

Wave atoms are a recent addition to the repertoire of mathematical transforms of computational harmonic analysis. They come either as an orthonormal basis or a tight frame of directional wave packets, and are particularly well suited for representing oscillatory patterns in images. They also provide a sparse representation of wave equations, hence the name wave atoms [1]. Magnetic Resonance Imaging (MRI) is a notable medical imaging technique that has proven to be particularly valuable for examination of the soft tissues in the body. MRI is primarily used to demonstrate pathological or other physiological alterations of living tissues and is a commonly used form of medical imaging. Because of the resolution of MRI and the technology being essentially harmless it has emerged as the most accurate and desirable imaging technology [2]. Despite significant improvements in recent years, magnetic resonance (MR) images often suffer from low Signal to Noise Ratio (SNR) especially in brain imaging. This paper presents an improved multi resolution de-noising method to de- noise Magnetic Resonance Images using Wave Atom Shrinkage, histogram based noise variance estimation [3] and modified threshold calculation that leads to the improvement of SNR in high noise level images.

The paper is organized with sections as follows. In section 2, the work related to this paper is briefly explained, Section 3 briefly explained about the rician noise which is usually present in MRI, Section 4 deals with the explanation of the estimation of rician noise variance, used in this method, In section 5, the theoretical concepts of wave atom transforms is described, in section 6, the application of wave atom transform and wavelet transforms to MRI and observations are discussed. In section 7, the paper is concluded by briefly explained the pros and cons of the proposed method.

## II. RELATED WORK

The image processing literature presents a variety of de-noising methods. Many of the popular de-noising algorithms suggested are based on wavelet thresholding [4]–[7]. These approaches attempt to separate significant features from noise in the frequency domain and simultaneously preserve them while removing noise. If the wavelet transform is applied on MR magnitude data directly, both the wavelet and the scaling coefficients of a noisy MRI image become biased estimates of their noise-free counterparts. Therefore, it was suggested [5] that the application of the wavelet transform on squared MR magnitude image data (which is noncentral chi-square distributed) would result in wavelet coefficients no longer being biased estimates of their noise-free counterparts. Although the bias still remains in the scaling coefficients, it is not signal-dependent and can therefore be easily removed [5], [7]. The difficulty with wavelet or anisotropic diffusion algorithms is again the risk of over-smoothing fine details particularly in low SNR images [8]. From these points, it is understood that all the algorithms have the drawback of over-smoothing fine details. In [9], stated that oscillatory functions or oriented textures have a significantly sparser expansion in wave atoms than in other fixed standard representations like Gabor filters, wavelets and curvelets. In [10], denoising using Wave Atom is done by estimating the noise variance by trial and error method. In [11], denoising using Wave Atom is done by estimating the noise variance by histogram technique.

## III. RICIAN NOISE

The image intensity in magnetic resonance magnitude images in the presence of noise is to be governed by a Rician distribution. Rician noise depends on the data itself, it is not additive, so to add Rician noise to data, what we really mean is make the data Rician distributed [12],[13]. The magnetic resonance signals are acquired in quadrature channels. Each signal produces an image that is degraded by a zero-mean Gaussian noise of standard deviation  $\sigma_0$  (which we define as the noise level). The two images are then combined into a magnitude image and the Gaussian noise PDF is transformed into a Rician noise PDF. The joint probability density of the noise from two quadrature channels can be expressed as [14]:

$$p(r, i) = \frac{1}{2\pi\sigma_0} \exp\left\{-\frac{r^2 + i^2}{2\sigma_0^2}\right\} \quad (1)$$

The expectation values for the mean magnitude and the variance are [12]:

$$I = \sigma_0 \sqrt{\frac{\pi}{2}} \exp\left\{-\frac{X^2}{4\sigma_0^2}\right\} \left[ 1 + \frac{X^2}{2\sigma_0^2} I_0\left(\frac{X^2}{4\sigma_0^2}\right) + \frac{X^2}{2\sigma_0^2} I_1\left(\frac{X^2}{4\sigma_0^2}\right) \right] \quad (2)$$

$$\sigma^2 = X^2 + 2\sigma_0^2 - \frac{\pi\sigma_0^2}{2} \exp\left\{-\frac{X^2}{4\sigma_0^2}\right\} \left[ \frac{X^2}{2\sigma_0^2} I_0\left(\frac{X^2}{4\sigma_0^2}\right) + \frac{X^2}{2\sigma_0^2} I_1\left(\frac{X^2}{4\sigma_0^2}\right) \right] \quad (3)$$

where  $I_0$  and  $I_1$  are modified Bessel functions of the first kind and  $X$  denote the MR magnitude image.

## IV. NOISE VARIANCE ESTIMATION

Noise variance estimation plays an important role for the proper selection of threshold in the multi-resolution techniques. Many noise variance estimation methods are available in the literature; here the Automatic estimation of

the noise variance from the histogram of an MR image developed in [3] is used. This method is superior in terms of the mean squared error. Let  $\{l_i\}$  with  $i = 0, \dots, K$  denote the set of boundaries of histogram bins. Furthermore, let  $n_i$  represent the number of observations (counts) within the bin  $[l_{i-1}, l_i]$ , which are multinomially distributed. Then, the joint PDF of the histogram data is given by :

$$p(\{n_i\} / \sigma, \{l_i\}) = \frac{N_K!}{\prod_{i=1}^K n_i!} \prod_{i=1}^K \left( \frac{l_i - l_{i-1}}{\sigma} \right)^{n_i} \exp\left\{-\frac{m^2}{2\sigma^2}\right\} \quad (4)$$

with  $N_K = \sum_{i=1}^K n_i$  the total number of observations within

the Partial histogram and  $p_i$  the probability that an Observations within the partial the range  $[l_{i-1}, l_i]$ . For Rayleigh distributed observations, this probability is given by

$$p_i(\sigma) = \frac{\int_{l_{i-1}}^{l_i} \frac{m}{\sigma} \exp\left\{-\frac{m^2}{2\sigma^2}\right\} dm}{\sum_{i=1}^K \int_{l_{i-1}}^{l_i} \frac{m}{\sigma} \exp\left\{-\frac{m^2}{2\sigma^2}\right\} dm} \quad (5)$$

Since

$$\int_a^b \frac{m}{\sigma} \exp\left\{-\frac{m^2}{2\sigma^2}\right\} dm = e^{-\frac{a^2}{2\sigma^2}} - e^{-\frac{b^2}{2\sigma^2}} \quad (6)$$

Using (6), (5) simplifies to

$$p_i(\sigma) = \frac{e^{-\frac{l_{i-1}^2}{2\sigma^2}} - e^{-\frac{l_i^2}{2\sigma^2}}}{e^{-\frac{l_{i-1}^2}{2\sigma^2}} - e^{-\frac{l_i^2}{2\sigma^2}}} \quad (7)$$

If the set of observations  $\{n_i\}$  is fixed and  $\sigma$  is regarded as a variable, the joint PDF given in (4) is called a likelihood function. The ML estimate is then found by maximizing this likelihood function  $L$  with respect to  $\sigma$ :

$$\sigma_{ML, K} = \arg \max_{\sigma} L(\sigma / \{n_i\}, \{l_i\}) \quad (8)$$

Equivalently, the ML estimate of  $\sigma$  is found by minimizing  $-\ln L$  with respect to  $\sigma$ :

$$\sigma_{ML, K} = \arg \max_{\sigma} \left[ N_K \ln \left( e^{-\frac{l_0^2}{2\sigma^2}} - e^{-\frac{l_K^2}{2\sigma^2}} \right) - \sum_{i=1}^K n_i \ln \left( e^{-\frac{l_{i-1}^2}{2\sigma^2}} - e^{-\frac{l_i^2}{2\sigma^2}} \right) \right] \quad (9)$$

Eq. (9) is the ML estimate of the noise standard

deviation from  $K$  bins. This result can be interpreted as follows. The joint PDF (4) with the ML estimate (9) as parameter generates the set of observations (counts) from which this parameter is estimated with a larger probability than a joint PDF with any other value of  $\sigma$ . For implementation software, we refer to the homepage <http://visielab.ua.ac.be/staff/poot/BackgroundNoiseLvlEst.z>.

## CONCLUSIONS

A better scheme is presented for the denoising of magnetic resonance imaging using wave atom transform. It is proved that the proposed threshold provides a better quality on MRI as compared to old threshold. The edge preserving property is clearly an advantage of the proposed method.

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