



A Study of High Embedding Rate Steganography Algorithm for Color Images using Complex Wavelet Transforms

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Abstract— *The basic requisites for any steganography algorithm include imperceptibility, embedding capacity and robustness to communication attacks. To measure the quality of gray-scale or color image, the metric such as PSNR is widely used by researchers in steganography. However, for digital color image, there are many other options such as WPSNR, $PSNR_E$, $PSNR_Y$, PSNR-HSR, CQM etc. In this paper, we have presented a comparative study of transforms such DD DWT and DD DT DWT based high rate steganography algorithm using the Fusion method. The analysis of algorithm is done using different imperceptibility metrics, such as $PSNR_Y$, and CQM rather than PSNR, structural similarity metric, SSIM, provable security metric, KLDiv and histogram Through experimental results, it is observed that DD DT DWT outperforms than the other transform in terms of imperceptibility, structural similarity and provable security.*

Keywords— DWT; CWT; DD DWT; DD DT DWT; PSNR; $PSNR_Y$; CQM; SSIM; KLDiv

I. INTRODUCTION

The basic requisite for any steganography algorithm include imperceptibility, high rate embedding and robustness to noise that may occur due to communication channel. For high rate embedding steganography algorithm, Tolba and Ghonemy [21] proposed the Wavelet Fusion method. The discrete wavelet transform (DWT) has been used widely in number of fields such as image compression, image de-noising, image restoration, digital data hiding, watermarking and steganography. But, due to the lacking of properties such as shift invariance and significant directional selectivity, researchers have explored other transforms also such as Wavelet-like transform (viz., Slantlet transform (SLT)), Contourlet transform (CTT), Complex wavelet transforms (CWT) etc.

Thompson et al [20] showed the superiority of complex wavelets over discretely sampled wavelets over the limits of watermark capacity, using established information theoretic methods. Li and Lv [6] have presented a new watermarking algorithm based on the DT-CWT and PCNN. Their experimental results show that their algorithm can combine the visual system and the embedding process perfectly to enhance the robustness and invisibility of the watermark.

Ali and Salman [1] proposed the image compression scheme using 2D Dual Tree Complex Wavelet Transform (2D DT CWT) and found that for certain value of threshold, their scheme give higher rate of compression and have lower RMS error compared with other methods based on DWT, DT-RWT and DCT. Srinivasulu Reddy et al. [19] have also proposed an image compression technique using 2D DT CWT the in combination with Huffman encoder and found higher compression ratio, good BPP and better PSNR.

DT DWT and DT CWT have also been proposed for image de-noising. Image de-noising is used to remove the noise from the cover image such that the important information of the cover is retained as much as possible. Chinnarao and Madhavilatha [3] proposed a new soft thresholding method for image de-noising using DT CWT and found less artifacts and ringing artifacts in the process of reconstruction of image. Shyam Lal et al. [17] have demonstrated that the complex DD DT DWT outperforms a number of other existing wavelet transform techniques. Kumar [8] have proposed DT-CWT based Block Non-linear de-noising and found that BlockShrink approach to DT-CWT is better than the BiShrink approach proposed by Selesnick [16].

The complex wavelet transforms have also been proposed in place of DWT for steganography schemes. Singh and Siddiqui [18] proposed a steganography technique based on complex wavelet transform, singular value decomposition and chaotic sequence which is shown to be more robust against common image processing and geometric attacks. Sathisha et al. [13] have proposed DT-CWT based high capacity steganography using coefficient replacement and adaptive scaling. The new concept of replacing HH sub band coefficients of DTCWT of cover image by LL sub band coefficients of payload is introduced to generate intermediate stego object. It is observed that the new coefficient replacement technique improves the security, PSNR and 100 percent hiding capacity. The adaptive scaling and use of DTCWT transformation in the proposed technique shows better results compared to the existing techniques. Kumar and Muttoo [7], [9], [10] have proposed high rate embedding steganography algorithm based on Wavelet Fusion method using DD DWT and DD DT DWT with combination of T-codes. They observed through experimental results that in terms of PSNR and embedding capacity, DD DT DWT outperforms to DWT. Kumar [7] has demonstrated that 2D DD DT DWT outperforms the other transforms such as Haar, CDF9/7, SLT, and CTT for high rate embedding steganography scheme for color images based on Wavelet Fusion method as the proposed algorithm provides higher PSNR/PSNR_v/CQM, almost perfect SSIM value and good provable security as compared to other transforms. The histogram analysis included to show the degradation of the quality of images- the cover image and the stego-image. Usually when it comes to show histogram of a color image, the comparison is provided by showing histograms each for Red, Green and Blue channels, separately. However, the author provides the histogram comparison between the original color image and the corresponding stego-image in combination of all three channels. Usually, when the histograms are compared separately for each channel, it is found that 2D DD DT DWT shows not significant visual changes between the cover image and the stego-image, however when the histograms for the combined channel are compared, the visual changes are not insignificant for some of the image formats.

In this paper, a high rate embedding steganography algorithm [21] based on the two Complex Wavelet Transforms, DD DT DWT and DD DWT is proposed and compared with the existing algorithm. For the imperceptibility measures, not only the PSNR, but also PSNR_v and CQM are used. Other metrics used for comparison between the proposed algorithm and the existing algorithm include the structural similarity metric, SSIM, the provable security metric, KLDiv and histograms. The Variable length source encoder, T-codes is used in place of Huffman encoding for obtaining the secret message to be embedded from the original message. It has been shown that T-codes are better option than the Huffman as according to Manoharan [11] who proposed the use of T-codes to encode messages prior to embedding the messages in the cover media, this system will be more tolerant to media transformations that result in some bit losses or bit inversions in the hidden message.

In the subsequent subsections, we summarize the complex Wavelet transforms proposed by Selesnick [5] and the Wavelet Fusion method [21].

1.1 DD DWT and DD DT DWT

The double-density DWT (DD DWT) developed by Selesnick [14] was motivated in part by the *undecimated* DWT. It is based on a single scaling function (low pass) and two distinct wavelets (high pass), where the two wavelets are designed to be offset from one another by one half—the integer translates of one wavelet fall midway between the integer translates of the other wavelet: $\Psi_2(t) = \Psi_1(t - 0.5)$.

The dual-tree DWT (DT DWT), also developed by Selenick [15] was motivated by the special properties of complex wavelet transforms. The DT DWT is based on concatenating two critically sampled DWTs. The filterbank structure corresponding to the Dt DWT simply consists of two critically sampled iterated filterbanks operating in parallel. The performance gains provided by the DT DWT come from designing the filters in the two filterbanks appropriately.

The double density dual-tree DWT (DD DT DWT) is a discrete wavelet transform that combines the double-density DWT and the dual-tree DWT, each of which has its own characteristics and advantages. It is based on two distinct scaling functions and four distinct wavelets $\Psi_{h,i}(t)$ and $\Psi_{g,i}(t)$ $i = 1, 2$, where the two wavelets are offset from one another by one half, as is : $\Psi_{h,1}(t) = \Psi_{h,2}(t-0.5)$; $\Psi_{g,1}(t) = \Psi_{g,2}(t- 0.5)$ and where the two wavelets $\Psi_{h,1}(t)$ and $\Psi_{h,2}(t)$ form an approximate Hilbert transform pair, as do $\Psi_{g,1}(t)$ and $\Psi_{g,2}(t)$.

The filterbank structure corresponding to the DD DT DWT consists of two oversampled iterated filterbanks operating in parallel, similar to the dual-tree DWT. The oversampled filter bank is illustrated in Fig. 1.1. The

iterated oversampled filter bank, corresponding to the implementation of the double-density dual-tree, is illustrated in Fig. 1.2. We denote the filters in the first filterbank by $h_i(n)$ and the filters in the second filterbank by $g_i(n)$, for $i=0,1,2$.

Both DD DWT and DD DT DWT are over complete by a factor of two, nearly shift-invariant and are based on FIR perfect reconstruction filter banks, but they are quite different from one another in other important respects (see the table 1.1).

Table 1.1 Differences between the double-density DWT and the dual-tree DWT [15]

DT DWT	DD DWT
The two wavelets form an approximate Hilbert transform pair	The two wavelets are offset by one half.
There are fewer degrees of freedom for design (achieving the Hilbert pair property adds constraints)	There are more degrees of freedom for design.
Different filterbank structures are used to implement the dual-tree DWTs	Different filterbank structures are also used to implement the double-density DWTs
The dual-tree DWT can be interpreted as a complex-valued wavelet transform, which is useful for signal modeling and denoising	The double-density DWT cannot be interpreted as such
The dual-tree DWT can be used to implement 2-D transforms with directional Gabor-like wavelets, which is highly desirable for image processing	The double-density DWT cannot be, although it can be used in conjunction with specialized post-filters to implement a complex wavelet transform with low-redundancy

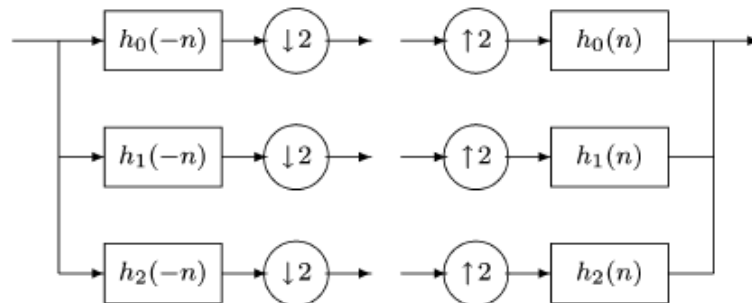


Fig. 1.1 Oversampled analysis and synthesis filterbank.

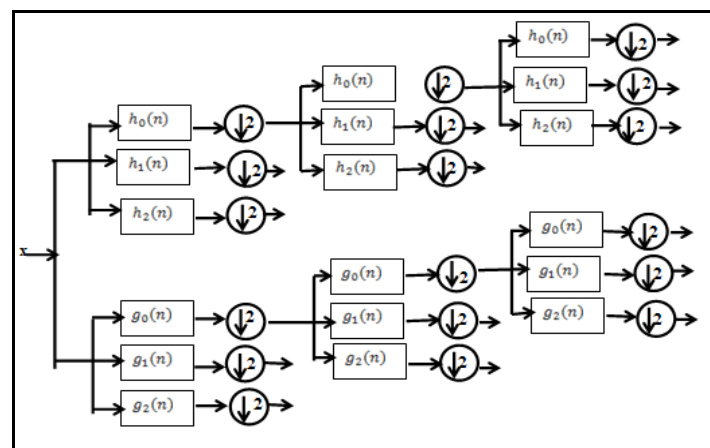


Fig. 1.2 Iterated filterbank for the double-density dual-tree DWT.

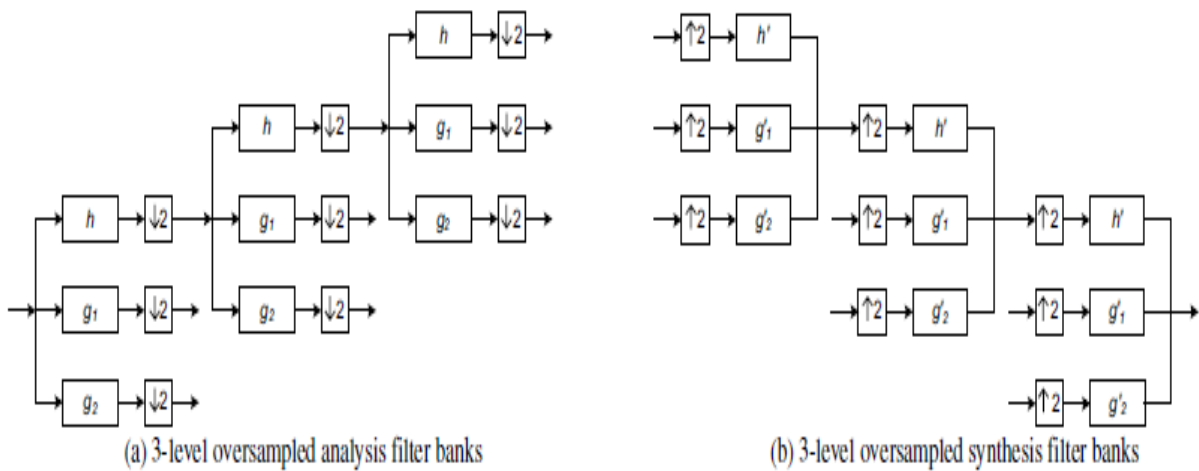
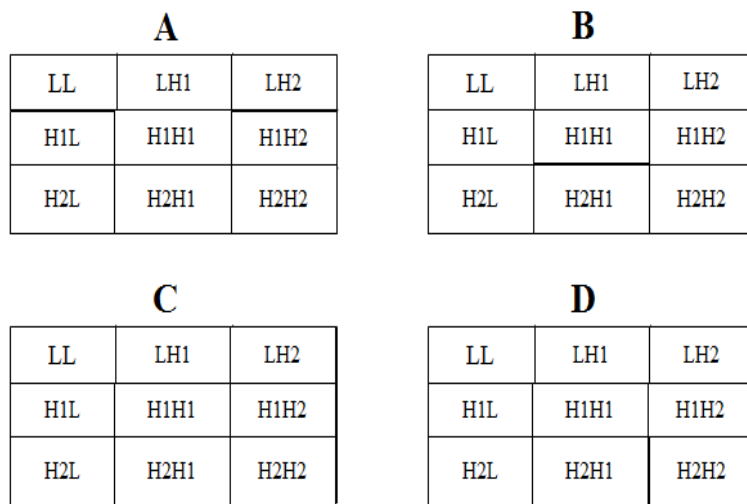


Fig.1.3 (a) 3-level oversampled analysis filter bank of DD DWT. (b) 3-level oversampled synthesis filter bank.[24]

The 2-D DD DWT can be implemented by tree structured filter banks, where the tree propagation is done on the *LL* sub-band image of each stage as shown in Fig. 1.3(a). At each decomposition level, the input signal is decomposed by one low-pass (scaling) filter and two high-pass (wavelet) filters, following the down sampled by 2. In the synthesis process as shown in Fig.1.3(b) three-channel filter banks are inverted using perfect reconstruction with FIR filters.

There are two types of 2-D DD DT DWT: The 2-D DD DT real-oriented DWT and 2-D DD DT complex-oriented DWT. The first one is 2-times expansive and second is 4-time expansive to DD DWT. The 2-D DD DT Real DWT of a 2-D image is implemented by using two oversampled 2-D DD DWTs in parallel as shown in Fig. 1.2. Then, for each pair of sub-bands we take the sum and difference, whereas 2-D DD DT Complex DWT gives rise to twice as many wavelets in the same dominating orientations as the 2-D DD DT Real DWT. This transform is implemented by applying four 2-D DD DWTs in parallel to the same input data with distinct filter sets for the rows and columns. We then take the sum and difference of the sub-band images. This results into 32 oriented wavelets, as shown in Fig. 1.4(a) and the one part is shown for the image ‘Zoneplate.png’ in Fig. 1.4(b).



(a)

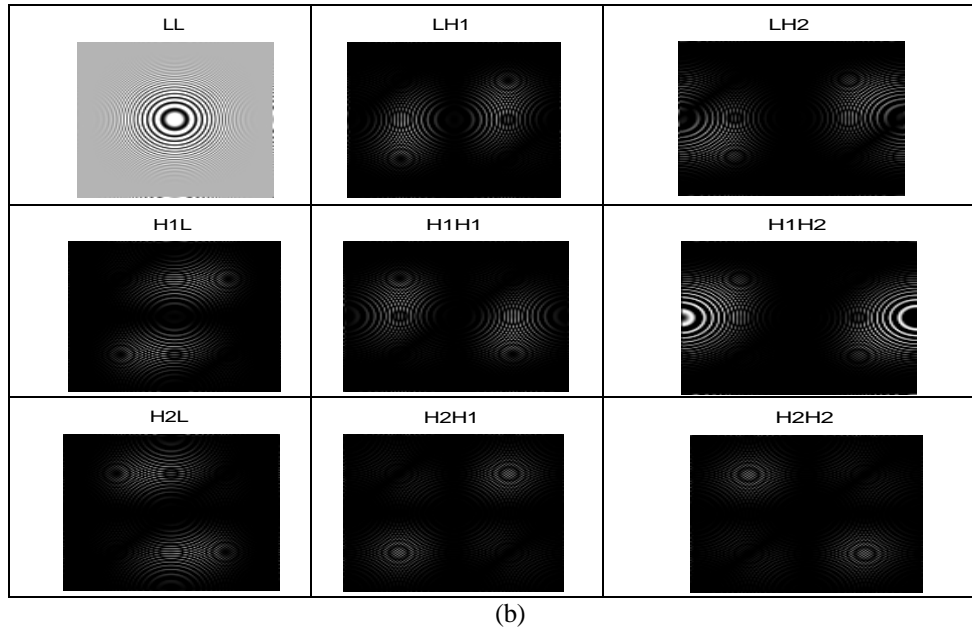


Fig. 1.5 (a) 1-stage decomposition of 2-D DD DT Complex DWT [15] (b) Part A demonstrated for the image 'Zoneplate.png'

1.2 Wavelet Fusion Method

The Wavelet fusion method is the high bit rate data hiding proposed by Tolba, Ghonemy, Taha & Khalifa [21]. Since the ordinary wavelet filters have floating point coefficients, a normalization operation is applied on the cover image so that the wavelet coefficients are converted in the range of 0.0 and 1.0. The secret message is converted to binary bits. The fusion technique follows the formula:

$$f^*(x, y) = f(x, y) + \alpha * g$$

where f^* is the modified DWT coefficient, f is the normalised wavelet coefficient, g is the secret message bits and alpha (α) is the embedding strength which ranges from 0.0 to 1.0.

If the secret message bit is 1 the +alpha is added to the cover image wavelet coefficient and if it is 0 the -alpha is added to the wavelet coefficient of the cover image. The embedding is applied on each color plane separately. To overcome the problem of overflow or underflow, the cover's normalized pixels are adjusted before the embedding process takes place so that the reconstructed pixels do not go out of range.

In section II, we explain the proposed algorithm and section III contains the experimental results and their analysis. The conclusion is given in section IV.

II. PROPOSED ALGORITHM

In the following paragraphs the embedding method is presented.

To recover the original message, we use T-encoder in place of Huffman encoder. T-codes provides self-synchronization at the decoding stage. The high frequency sub-bands are obtained from the cover image using the CWT. The 1-stage of 2D DD DT DWT provides 8-high sub-bands, namely, $LH_1, LH_2, H_1 L, H_1 H_1, H_1 H_2, H_2 L, H_2 H_1$ and $H_2 H_2$, whereas the 1-stage of 2-D DD DWT give 12 high sub-bands, when applied to the cover image. We have embedded the binary bits of secret data using Fusion technique in the first 3-high sub-bands only, however, embedding can be done in all the high frequency sub-bands and in more than 1-stage of CWT and thus high embedding rate can be obtained.

2.1 EMBEDDING PROCEDURE

Steganography embedding procedure [9] can be described using the following steps.

1. First normalize the color image by choosing alpha, preferably =0.05 and reconstruct the pixels values that lie in interval [alpha, 1-alpha].
2. Apply 1-stage of 2D DD DT real DWT or 2d DD DWT to obtain 8 high subbands for each color plane of image.
3. Obtain the binary form of the data to be embedded using the T-codes.

4. Generate pseudorandom permutation, using a stego-key, of the size of sub-band.
5. Enter the number of times the secret message to be embedded
6. Embed the message by adding or subtracting the alpha in the pixels according to message bit is 1 or 0.
7. Apply inverse of 2-D DD DT real DWT on each color plane separately.
8. Obtain the stego image by normalizing the resultant image.

2.2 EXTRACTION PROCEDURE

The extraction procedure [9] can be described using the following steps.

1. Apply 2-D DD DT real DWT on each color plane of the stego-image.
2. Using the pseudorandom permutation based on the stego-key, Obtain the selected embedded coefficients.
3. Obtain the secret message bit ‘s’ by subtracting the transformed coefficients of original image from the transformed stego image coefficients obtained by using 2-D DD DT real DWT.
4. If the value of ‘s’ is positive, embedded bit is 1 and the embedded bit is 0 if s is negative.
5. Decode the secret message into the original message using T-decoding.

The Fig. 1.6 illustrates the embedding and extraction schemes.

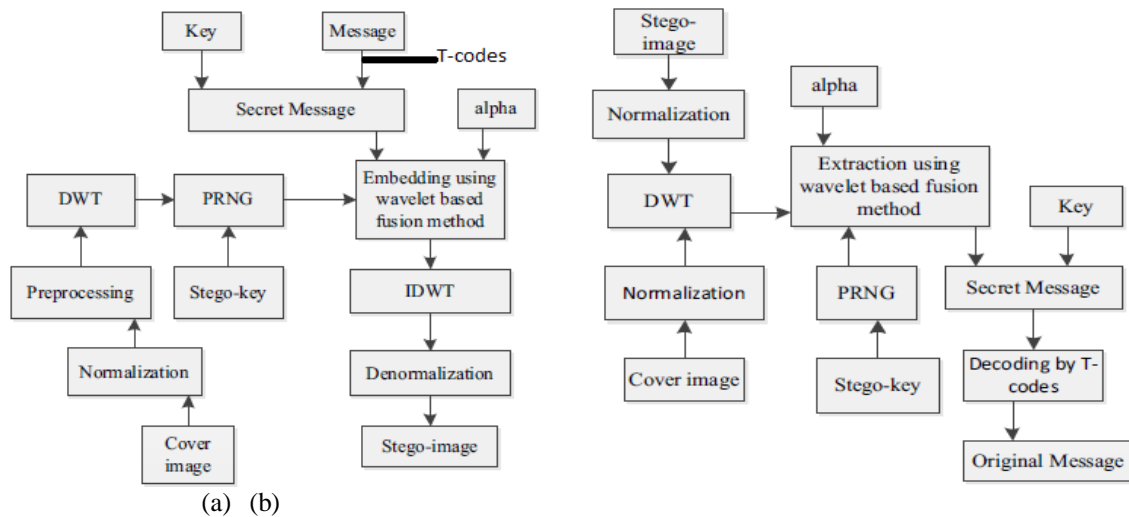


Fig. 1.6 (a) Embedding Process (b) Extraction process

III. EXPERIMENTAL RESULTS

We have implemented the proposed Algo 2.1 using number of test images, results of the some of the images listed here are shown in Fig. 3.1. The embedding capacity used varying from low 500 bytes to high 6000 bytes. Experiments have been performed using MATLAB 7.0. In the consequent subsection we summarize the metrics used for comparison of the proposed algorithm and the existing algorithm

A. Imperceptibility

According to Mathworks [25], the PSNR for color images can be computed by first converting the image to a color space that separates the intensity (luma) channel, such as $YCbCr$, and then the PSNR can be obtained only on the luma channel, (denoted by $PSNR_Y$). The recommendation for this approach is suggested because the human eye is most sensitive to luma information, the Y (luma), in $YCbCr$ represents a weighted average of R, G and B and G is given the most weight, again because the human eye perceives it most easily.

Another simple and effective full-reference color image quality measure (CQM), proposed by Yalman and Erturk [23], is based on reversible luminance and chrominance (YUV) color transformation and peak signal-to-noise ratio (PSNR) measure. The CQM value is given as:

$$CQM = (PSNR_Y \times R_w) + \left(\frac{PSNR_U + PSNR_V}{2} \right) \times C_w,$$

where the CQM is composed of the weighted luminance quality measure ($PSNR_Y \times R_w$) and weighted color quality measure ($PSNR_U + PSNR_V$) $\times C_w$ components. C_w is the weight on the human perception of the cones and R_w , is the weight on the human perception of the rods. The values of C_w and R_w are calculated as 0.0551

and 0.9449, respectively. The notation PSNR used for color images will mean the average value of PSNR calculated for each of the Red, Green and Blue channels.

B. Structural Similarity

The structural similarity (SSIM) [22] is composed of three values: Luminance comparison; Contrast comparison; and Structural comparison. These components are normalized such that they are 1.0 for identical images. The SSIM index is the product of these three components (raised by an exponent, if required). The formula for this measure is given by

$$SSIM(E, F) = \frac{(\mu_x \mu_y + C_1) (2\sigma_{xy} + C_2)}{(\mu_x^2 + \mu_y^2 + C_1) (\sigma_x^2 + \sigma_y^2 + C_2)}$$

where $\mu_x, \mu_y, \sigma_x, \sigma_y$ and σ_{xy} local statistics parameters of the two images E and F and C_1, C_2 are constants used to make it finite.

The SSIM index models any distortion as a combination of three different factors: loss of correlation, luminance distortion and contrast distortion. The mean similarity measure MSSIM varies in the interval [-1, 1]. The best value 1 is achieved if and only if E=F.

C. Provable Security

According to Cachin [2], a steganography algorithm is ϵ -secure if the relative entropy (also called K-L divergence) between distributions of cover object and stego-object is less than or equal to ϵ . Let random variable C and S denote the cover image and stego image respectively and let P_C and P_S represent the probability mass functions (pmfs) of C and S, respectively. The K-L divergence between these two pmfs, P_C and P_S , is defined as:

$$D(P_C//P_S) = \sum_{g \in G} [P_C(g) \log(P_C(g)/P_S(g))]$$

where $g \in G \approx \{0, 1, 2, \dots, 255\}$ is the pixel value in grayscale images.

The stego system is considered perfectly secure in the Cachin's sense if $D(P_C//P_S) = 0$.



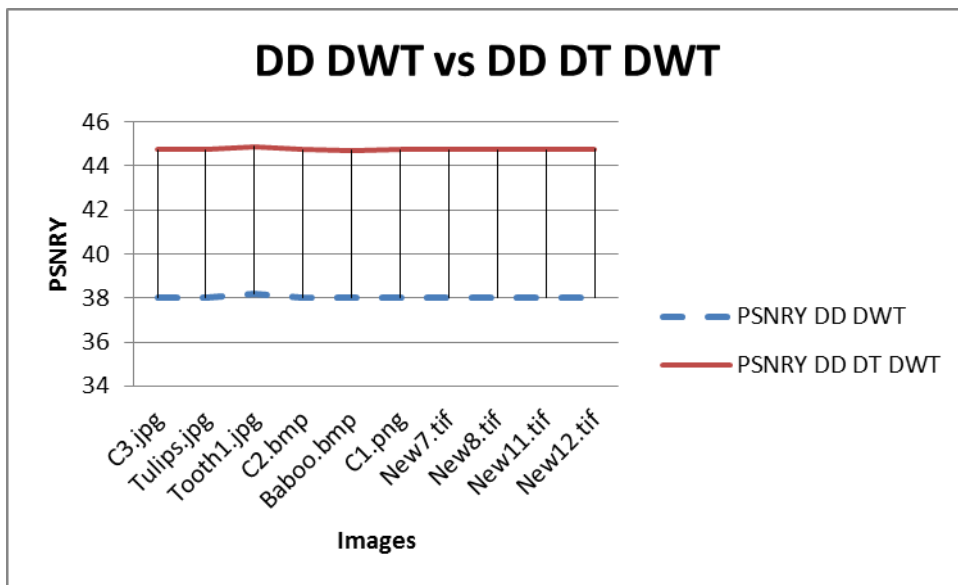
Fig. 3.1 Test Images

The results of imperceptibility measures such as $PSNR_Y$, and CQM for Algo 2.1 with embedding capacity = 4000 bytes are shown in the Table 3.1.

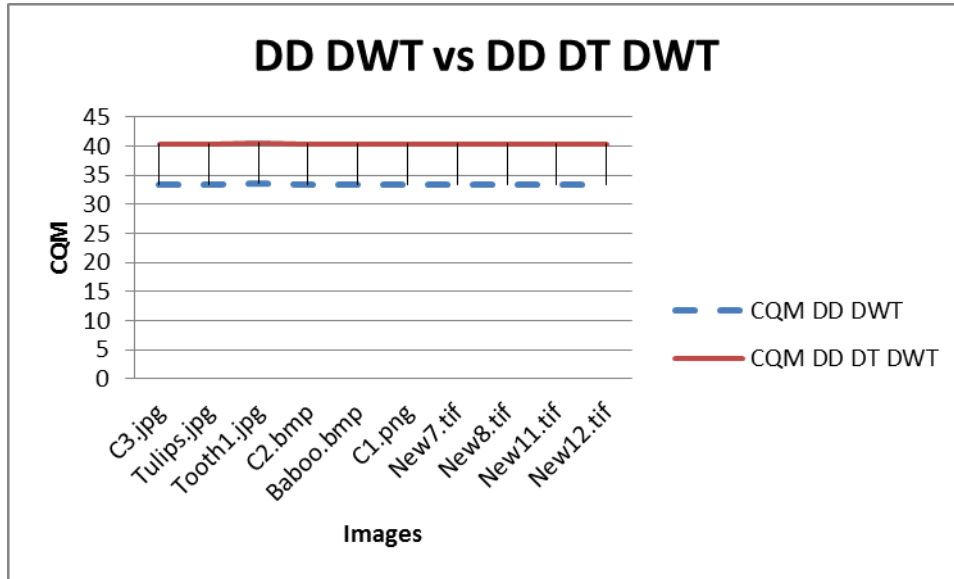
From the Figure 3.2, it is observed that DD DT DWT based Algo 2.1 improves imperceptibility of the image in each of the measures of imperceptibility, $PSNR_Y$ and CQM, used for the evaluation, assuming $\alpha = 0.05$ and N, number of embedding=5. In fact as the values of alpha is lowered, higher the $PSNR$ values expected, though sacrificing the embedding rate.

Table 3.1: $PSNR_Y$ and CQM values for Algo 2.1 based on DD DWT and DD DT CWT with capacity= 4000, $\alpha=0.05$ and No. of embedding=5

Image	$PSNR_Y$		CQM	
	DD DWT	DDDT DWT	DD DWT	DDDT DWT
C3.jpg	38.013859	44.754529	33.272689	40.345774
Tulips.jpg	38.019965	44.746557	33.332581	40.348409
Tooth1.jpg	38.182441	44.876404	33.480516	40.515707
C2.bmp	38.008482	44.734869	33.261721	40.339351
Baboo.bmp	37.999806	44.724724	33.265558	40.339568
C1.png	38.000082	44.73502	33.259316	40.339351
New7.tif	38.027834	44.740815	33.28849	40.359132
New8.tif	38.01828	44.738084	33.27612	40.353027
New11.tif	38.031939	44.749963	33.312556	40.39937
New12.tif	38.027006	44.770396	33.277464	40.360016



(a)



(b)

Fig. 3.2 (a) PSNR_v and (b) CQM values of stego-images obtained using DD DWT and DD DT DWT

Table 3.2: SSIM values for Algo 2.1 based on DD DWT and DD DT DWT

Image	SSIM	
	DD DWT	DD DT CWT
C3.jpg	0.928078	0.98573
Tulips.jpg	0.942749	0.988556
Tooth1.jpg	0.884262	0.974917
C2.bmp	0.890639	0.977552
Baboo.bmp	0.9695	0.994152
C1.png	0.940342	0.988284
Cat.png	0.93736	0.987752
New7.tif	0.928298	0.985684
New8.tif	0.97509	0.995308
New11.tif	0.945528	0.989308
New12.tif	0.90596	0.981246

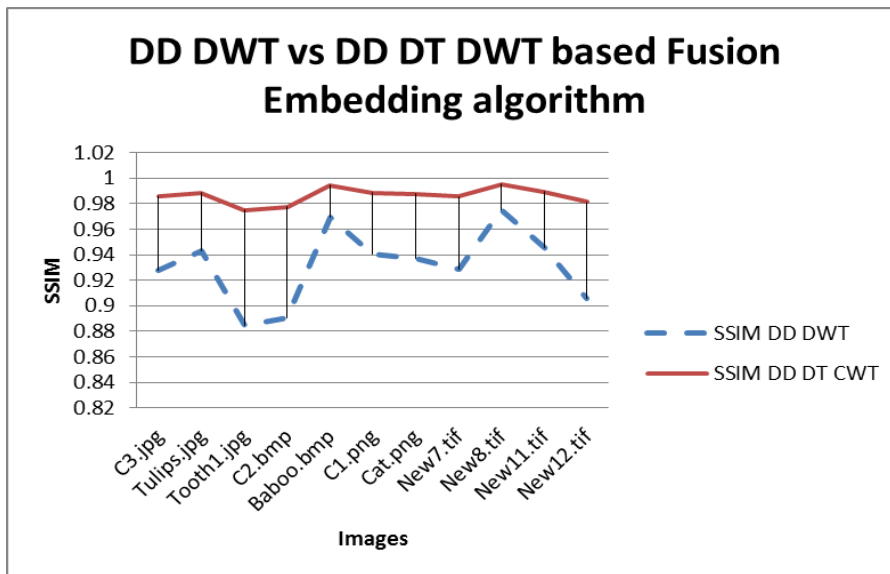


Fig.3.3 Comparison of DD DWT and DD DT DWT based Algo 2.1 in terms of SSIM

In Figure 3.3, we show the results obtained of structural similarity, SSIM, for the proposed algorithm 2.1 based on DD DWT and DD DT DWT using different image formats. It is observed that DD DT DWT based proposed algorithm gives almost perfect structural similarity than the other transform.

Table 3.4: KLdiv values for Algo 2.1 based on DD DWT and DD DT DWT

<i>Image</i>	<i>KLdiv</i>	
	<i>DD DWT</i>	<i>DD DT DWT</i>
<i>C3.jpg</i>	0.00055	0.000114
<i>Tulips.jpg</i>	0.000516	0.000109
<i>Tooth1.jpg</i>	0.000627	0.00013
<i>C2.bmp</i>	0.000301	0.000072
<i>Baboo.bmp</i>	0.000446	0.000091
<i>C1.png</i>	0.000447	0.000093
<i>Cat.png</i>	0.000385	0.000083
<i>New7.tif</i>	0.000882	0.000212
<i>New8.tif</i>	0.000734	0.000154
<i>New11.tif</i>	0.000743	0.000177
<i>New12.tif</i>	0.000406	0.000083

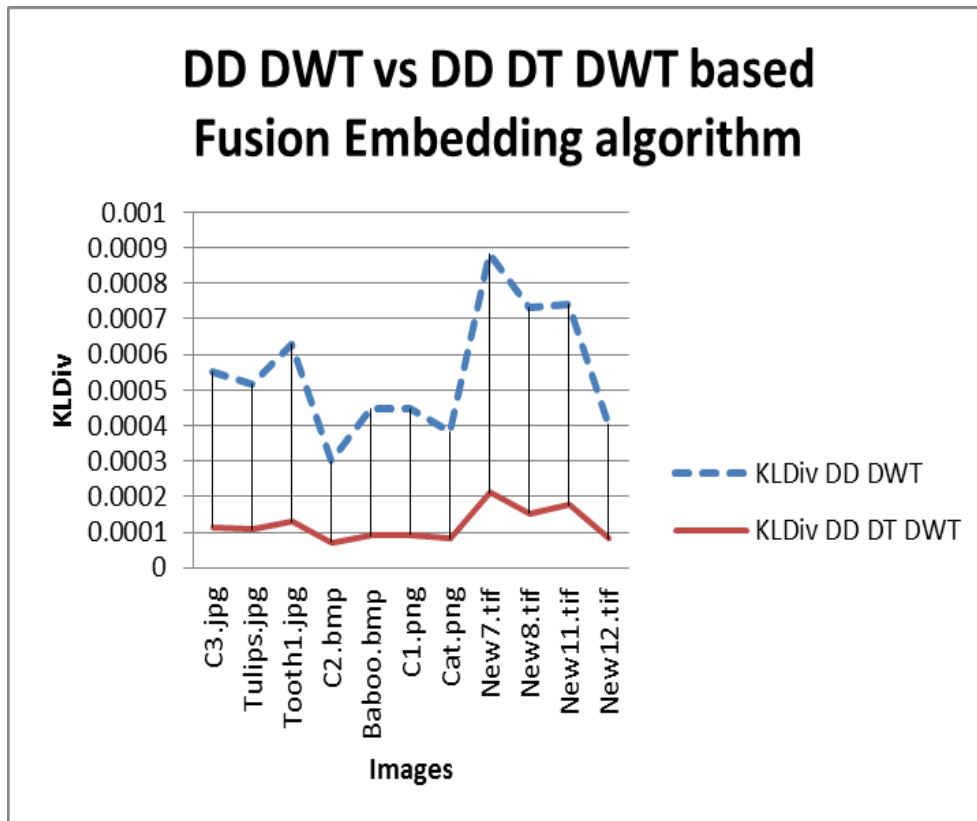


Fig.3.4 Comparison of DD DWT and DD DT DWT based Algo 2.1 in terms of KLdiv

In Figure 3.4, we show the results obtained of provable security, KLdiv, for the proposed algorithm 2.1 based on DD DWT and DD DT DWT using different image formats. It is observed that DD DT DWT based proposed algorithm provides better perfect provable security, as KLdiv is almost zero, as compared to the other transform.

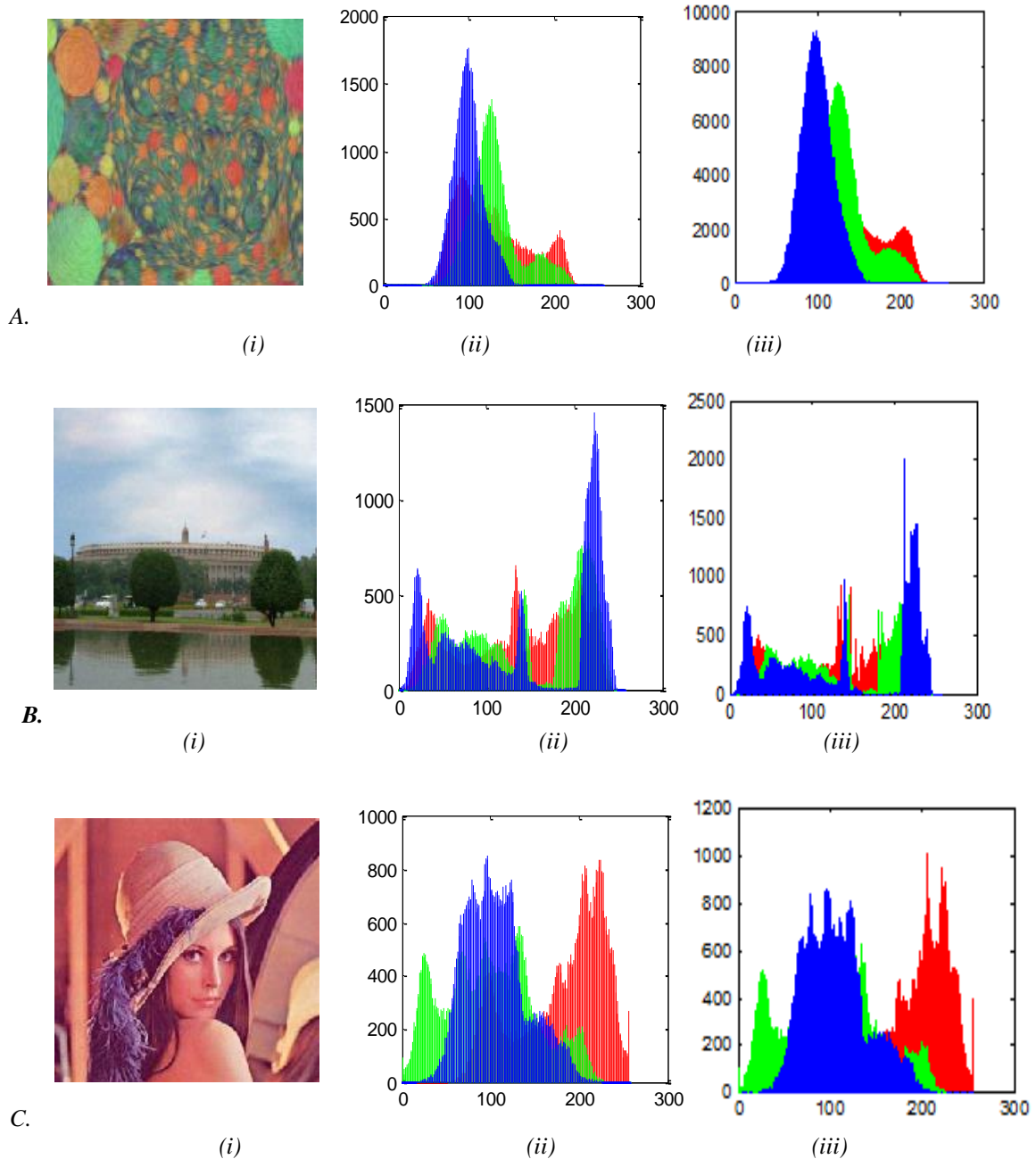


Figure 3.5 (i) Stego-image, (ii) corresponding histogram and (iii) histogram of original image for Algo 2.1
 D. C1.png; B. C2.bmp C. C3.jpg [7]

In Figure 3.5, we provide the stego images and the corresponding histograms obtained after the implementation of Algo 2.1 with respect to forward real DD DT real DWT for some of the images. It can be seen from the figures that DD DT real DWT based fusion method reasonably good against the statistical attack.

IV. CONCLUSIONS

This paper presents the study of performance of high embedding rate steganography algorithm using the wavelet Fusion method for color images based on different transforms, viz., DD DWT and Forward Real DD DT DWT. The secret message to be embedded is obtained from original text by applying self-synchronizing variable length codes- T-codes. It is observed that Forward Real DD DT DWT based Fusion method improves the results of PSNRY, CQM, SSIM and KLDiv and histogram analysis.

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