Online at www.ijcsmc.com

International Journal of Computer Science and Mobile Computing

A Monthly Journal of Computer Science and Information Technology

ISSN 2320–088X
IMPACT FACTOR: 6.199

IJCSMC, Vol. 8, Issue. 9, September 2019, pg.182 – 189

Numerical Simulation of the Coupled Dynamic Thermoelastic Problem for Orthotropic Bodies

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***Abstract:** The article considers the coupled dynamic thermoelasticity problem for a two-dimensional orthotropic material. A boundary value problem consists of the equations of motion and heat conduction attributing ut Party or, respectively, hyperbolic th and parabolic mu type in which the unknowns are the displacement I and temperature. Explicit and implicit difference schemes are compiled and solved numerically in two ways, and the coincidence of the numerical results is shown.*

***Keywords:** Thermo-elasticity, coupled problem, thermal conductivity, difference equations, explicit scheme, implicit scheme, grid method, elimination method.*

1. Introduction

The study of thermoelastic states of structures and their elements is an urgent problem of mathematical modeling. One of the main trends in the development of modern technology is the widespread use for the manufacture of various designs of composite materials consisting of structural components with various thermo-mechanical properties. The effectiveness of design solutions largely depends on the correct consideration of the thermo-mechanical behavior of the composite material under thermo-mechanical loads.

When formulating thermoelastic problems, one distinguishes between bound and unbound boundary value problems. In the general case, the coupled dynamic thermoelastic boundary-value problem consists of the equation of motion, which determines the Duhamel-Neumann relations, the Cauchy relation, and the heat influx equation with the corresponding initial and boundary conditions. Note that in this case, the equations of motion written in displacements and the heat influx equation are related, i.e. temperature as an unknown function enters into the equations of motion, and the heat flux equation depends on displacement. The coupled thermodynamic problem, firstly was considered by Biot [1] in 1956. Further, these studies were continued in the works of Lord-Shulman (1967) [2], Muller (1972) [3], Youssef (2006) [4], Aboudi (1985) [5] and others.

If the external factors causing the body motion change very slowly in time, then the inertial terms can be neglected in the equation of motion, treating the problem as quasistatic [6] . In this case, the initial conditions with respect to displacements disappear, but the quasistatic problem remains connected. If the values causing deformation and temperature change rather slowly from zero to their final values and remain in this state, then we will get a static problem. The displacement and temperature become time-independent and are functions of the coordinates of the position of the points, and the terms containing the derivatives with respect to time disappear in the equations. In this case, we have an unrelated problem of thermoelasticity [6].

2. Formulation of the Problem

Consider the related problem of thermoelasticity for anisotropic bodies, it consists of the equation of motion

$$\sum_{j=1}^{3}\sigma_{ij,j}+X_i=\rho\frac{\partial^2 u_i}{\partial t^2} \tag{1}$$

Duhamel-Neumann relations for anisotropic bodies

$$\sigma_{ij}=C_{ijkl}\varepsilon_{kl}-\beta_{ij}(T-T_0)\delta_{ij} \tag{2}$$

Cauchy relations

$$\varepsilon_{ij}=\frac{1}{2}\left(u_{i,j}+u_{j,i}\right) \tag{3}$$

heat equation for anisotropic materials

$$\lambda_{ij}T_{,ij}-c_\varepsilon\dot{T}-T\cdot\beta_{ij}\cdot\dot{\varepsilon}_{ij}=0 \tag{4}$$

with corresponding initial

$$u_i\big|_{t=t_0}=\phi_i,\quad \dot{u}_i\big|_{t=t_0}=\psi_i,\quad T\big|_{t=t_0}=T_0 \tag{5}$$

and boundary conditions

$$u_i\big|_{\Sigma_1}=u_i^0,\quad T\big|_{\Sigma}=\bar{T}_0,\quad \sum_{j=1}^{3}\sigma_{ij}n_j\Big|_{\Sigma_2}=S_i^0 \tag{6}$$

where, $\sigma_{ij}-$ stress tensor, $\varepsilon_{ij}-$ strain tensor, u_i- displacement, $T-$ temperature, X_i- volume force, $C_{ijkl}-$ fourth-rank tensor determining the mechanical properties of the material, $c_\varepsilon-$ heat at a constant deformation, $\beta_{ij}-$thermal expansion tensor, $\lambda_{ij}-$ heat flux tensor, $\rho-$ density of the body, $\delta_{ij}-$ Kronecker symbol.

In order to write the equation of motion in displacements, substituting eq. (3) into eq. (2), and obtained in eq. (1) in the two-dimensional case, we obtain:
equations of motion

$$\begin{cases} C_{1111}\dfrac{\partial^2 u}{\partial x^2}+\left(C_{1122}+C_{1212}\right)\dfrac{\partial^2 v}{\partial x\partial y}+C_{1212}\dfrac{\partial^2 u}{\partial y^2}-\beta_{11}\dfrac{\partial T}{\partial x}=\rho\dfrac{\partial^2 u}{\partial t^2} \\ C_{2222}\dfrac{\partial^2 v}{\partial y^2}+\left(C_{1212}+C_{2211}\right)\dfrac{\partial^2 u}{\partial x\partial y}+C_{1212}\dfrac{\partial^2 v}{\partial x^2}-\beta_{22}\dfrac{\partial T}{\partial y}=\rho\dfrac{\partial^2 v}{\partial t^2} \end{cases} \tag{7}$$

and 2D heat equations

$$\lambda_{11}\frac{\partial^2 T}{\partial x^2}+\lambda_{22}\frac{\partial^2 T}{\partial y^2}-c_\varepsilon\frac{\partial T}{\partial t}-T(\beta_{11}\frac{\partial^2 u}{\partial x\partial t}+\beta_{22}\frac{\partial^2 v}{\partial y\partial t})=0 \tag{8}$$

with initial

$$u\left(x,y,t\right)\big|_{t=0}=\phi_1,\ \frac{\partial u}{\partial t}\Big|_{t=0}=\psi_1,\ v\left(x,y,t\right)\big|_{t=0}=\phi_2,\ \frac{\partial v}{\partial t}\Big|_{t=0}=\psi_2,\ T\left(x,y,t\right)\big|_{t=0}=T_0$$

and boundary conditions in 2D case

$$u(x,y,t)\big|_{x=0}=u_0,\ u(x,y,t)\big|_{x=\ell_1}=\bar{u}_0,\ u(x,y,t)\big|_{y=0}=u_0',\ u(x,y,t)\big|_{y=\ell_2}=\bar{u}_0',\ v(x,y,t)\big|_{x=0}=v_0$$

$$,\ v(x,y,t)\big|_{x=\ell_1}=\bar{v}_0\ ,$$

$$v(x,y,t)\big|_{y=0}=v_0'\ ,\ v(x,y,t)\big|_{y=\ell_2}=\bar{v}_0',\ T(x,y,t)\big|_{x=0}=T_1(t),\ T(x,y,t)\big|_{x=\ell 1}=T_2(t),$$

$$T(x,y,t)\big|_{y=0}=T_1'(t)\ ,$$

$$T(x,y,t)\big|_{y=\ell_2}=T_2'(t)$$

3. Numerical Solution

Replacing the derivatives in equations (7) and (8) with the corresponding difference relations, we obtain [7,8]

$$C_{1111}\frac{u^n_{i+1,j}-2u^n_{i,j}+u^n_{i-1,j}}{h_1^2}+\left(C_{1122}+C_{1212}\right)\frac{v^n_{i+1,j+1}-v^n_{i-1,j+1}-v^n_{i+1,j-1}+v^n_{i-1j-1}}{4h_1h_2}+ \\ +C_{1212}\frac{u^n_{i,j+1}-2u^n_{i,j}+u^n_{i,j-1}}{h_2^2}-\beta_{11}\frac{T^n_{i+1,j}-T^n_{i-1,j}}{2h_1}=\rho\frac{u^{n+1}_{i,j}-2u^n_{i,j}+u^{n-1}_{i,j}}{\tau^2} \tag{9}$$

$$C_{2222}\frac{v^n_{i,j+1}+2v^n_{i,j}+v^n_{i,j-1}}{h_2^2}+\left(C_{1212}+C_{2211}\right)\frac{u^n_{i+1,j+1}-u^n_{i-1,j+1}-u^n_{i+1,j-1}+u^n_{i-1,j-1}}{4h_1h_2}+ \\ +C_{1212}\frac{v^n_{i+1,j}-2v^n_{i,j}+v^n_{i-1,j}}{h_1^2}-\beta_{22}\frac{T^n_{i,j-1}-T^n_{i,j-1}}{2h_2}=\rho\frac{v^{n+1}_{i,j}-2v^n_{i,j}+v^{n-1}_{i,j}}{\tau^2} \tag{10}$$

$$\lambda_{11}\frac{T^n_{i+1,j}-2T^n_{i,j}+T^n_{i-1,j}}{h_1^2}+\lambda_{22}\frac{T^n_{i,j+1}-2T^n_{i,j}+T^n_{i,j-1}}{h_2^2}-c_\varepsilon\frac{T^{n+1}_{i,j}-T^n_{i,j}}{\tau}- \\ -T_0(\beta_{11}\frac{u^{n+1}_{i+1,j}-u^{n+1}_{i-1,j}-u^{n-1}_{i+1,j}+u^{n-1}_{i-1,j}}{4h_1\tau}+\beta_{22}\frac{v^{n+1}_{i,j+1}-v^{n+1}_{i,j-1}-v^{n-1}_{i,j+1}+v^{n-1}_{i,j-1}}{4h_2\tau})=0 \tag{11}$$

Solving the difference equations (9), (10) and (11) with respect to $u^{n+1}_{i,j}$, $v^{n+1}_{i,j}$ and $T^{n+1}_{i,j}$ accordingly, we obtain

$$u^{n+1}_{ij}=\frac{\tau^2}{\rho}(C_{1111}\frac{u^n_{i+1,j}-2u^n_{ij}+u^n_{i-1,j}}{h_1^2}+\left(C_{1122}+C_{1212}\right)\frac{v^n_{i+1,j+1}-v^n_{i-1,j+1}-v^n_{i+1,j-1}+v^n_{i-1,j-1}}{4h_1h_2}+ \\ +C_{1212}\frac{u^n_{i,j+1}-2u^n_{ij}+u^n_{i,j-1}}{h_2^2}-\beta_{11}\frac{T^n_{i+1,j}-T^n_{i-1,j}}{2h_1})+2u^n_{ij}-u^{n-1}_{ij} \tag{12}$$

$$v^{n+1}_{ij}=\frac{\tau^2}{\rho}(C_{2222}\frac{v^n_{i,j+1}+2v^n_{ij}+v^n_{i,j-1}}{h_2^2}+\left(C_{1212}+C_{2211}\right)\frac{u^n_{i+1,j+1}-u^n_{i-1,j+1}-u^n_{i+1,j-1}+u^n_{i-1,j-1}}{4h_1h_2}+ \\ +C_{1212}\frac{v^n_{i+1,j}-2v^n_{ij}+v^n_{i-1,j}}{h_1^2}-\beta_{22}\frac{T^n_{i,j-1}-T^n_{i,j-1}}{2h_2})+2v^n_{ij}-v^{n+1}_{ij} \tag{13}$$

$$T^{n+1}_{ij}=\frac{\tau}{c_\varepsilon}(\lambda_{11}\frac{T^n_{i+1,j}-2T^n_{ij}+T^n_{i-1,j}}{h_1^2}+\lambda_{22}\frac{T^n_{i,j+1}-2T^n_{ij}+T^n_{i,j-1}}{h_2^2}- \\ -T_0(\beta_{11}\frac{u^{n+1}_{i+1,j}-u^{n+1}_{i-1,j}-u^{n-1}_{i+1,j}+u^{n-1}_{i-1,j}}{4h_1\tau}+\beta_{22}\frac{v^{n+1}_{i,j+1}-v^{n+1}_{i,j-1}-v^{n-1}_{i,j+1}+v^{n-1}_{i,j-1}}{4h_2\tau}))+T^n_{ij} \tag{14}$$

As can be seen, equations (12) - (14) make it possible to find the values of displacements and temperature on the layer *(n +1)* if the values of displacements on the two previous layers are known. The values of displacements on the two initial layers *(n = 0, n = 1)* we will find from the initial conditions

$$u^0_{i,j}=\phi_1(x_i,y_j),\ v^0_{i,j}=\phi_2(x_i,y_j)\quad T^0_{i,j}=T_0 \tag{15}$$

we write equation (12) for *n = 0*

$$u_{i,j}^{1}=\frac{\tau^{2}}{\rho}(C_{1111}\frac{u_{i+1,j}^{0}-2u_{i,j}^{0}+u_{i-1,j}^{0}}{h_{1}^{2}}+\left(C_{1122}+C_{1212}\right)\frac{v_{i+1,j+1}^{0}-v_{i-1,j+1}^{0}-v_{i+1,j-1}^{0}+v_{i-1,j-1}^{0}}{4h_{1}h_{2}}+$$
$$+C_{1212}\frac{u_{i,j+1}^{0}-2u_{i,j}^{0}+u_{i,j-1}^{0}}{h_{2}^{2}}-\beta_{11}\frac{T_{i+1,j}^{0}-T_{i-1,j}^{0}}{2h_{1}})+2u_{i,j}^{0}-u_{i,j}^{-1} \quad (16)$$

Replacing the derivative in the initial condition $\dot{u}_{i}\big|_{t=t_0}=\psi_{i}$ by the difference relation, we obtain

$$\frac{u_{i,j}^{1}-u_{i,j}^{-1}}{2\tau}=\psi_{1}(x_{i},y_{j}) \quad \text{or} \quad u_{i,j}^{1}=2\tau\psi_{1}(x_{i},y_{j})+u_{i,j}^{-1} \quad (17)$$

Excluding values from equations (16) and (17) $u_{i,j,k}^{-1}$ we can find that

$$u_{i,j}^{1}=\frac{\tau^{2}}{2\rho}(C_{1111}\frac{u_{i+1,j}^{0}-2u_{i,j}^{0}+u_{i-1,j}^{0}}{h_{1}^{2}}+\left(C_{1122}+C_{1212}\right)\frac{v_{i+1,j+1}^{0}-v_{i-1,j+1}^{0}-v_{i+1,j-1}^{0}+v_{i-1,j-1}^{0}}{4h_{1}h_{2}}+$$
$$+C_{1212}\frac{u_{i,j+1}^{0}-2u_{i,j}^{0}+u_{i,j-1}^{0}}{h_{2}^{2}}-\beta_{11}\frac{T_{i+1,j}^{0}-T_{i-1,j}^{0}}{2h_{1}})+u_{i,j}^{0}+\tau\psi_{1}(x_{i},y_{j}) \quad (18)$$

The values of the functions *v* on the first layer are found in the same way . Replacing mixed derivatives with difference ratios shifted by indices, we can find relations for finding the temperature on the first layer i.e.

$$T_{i,j}^{1}=\frac{\tau}{c_{\varepsilon}}(\lambda_{11}\frac{T_{i+1,j}^{0}-2T_{i,j}^{0}+T_{i-1,j}^{0}}{h_{1}^{2}}+\lambda_{22}\frac{T_{i,j+1}^{0}-2T_{i,j}^{0}+T_{i,j-1}^{0}}{h_{2}^{2}}-$$
$$-T_{0}(\beta_{11}\frac{u_{i+1,j}^{1}-u_{i-1,j}^{1}-u_{i+1,j}^{0}+u_{i-1,j}^{0}}{2h_{1}\tau}+\beta_{22}\frac{v_{i,j+1}^{1}-v_{i,j-1}^{1}-v_{i,j+1}^{0}+v_{i,j-1}^{0}}{2h_{2}\tau}))+T_{i,j}^{0} \quad (19)$$

On the other layers $n=2,3,...$ values of displacements and temperature are respectively from equations (12), (13) and (14). All considered difference schemes were explicit, and the solution of which is calculated using recurrence relations.

To solve problem (1) - (6), an implicit scheme can be proposed. Why, in the first term of difference equation (9), replacing the superscript *n* by *n +1* , and the resulting scheme is implicit:

$$C_{1111}\frac{u_{i+1,j}^{n+1}-2u_{i,j}^{n+1}+u_{i-1,j}^{n+1}}{h_{1}^{2}}+\left(C_{1122}+C_{1212}\right)\frac{v_{i+1,j+1}^{n}-v_{i-1,j+1}^{n}-v_{i+1,j-1}^{n}+v_{i-1,j-1}^{n}}{4h_{1}h_{2}}+$$
$$+C_{1212}\frac{u_{i,j+1}^{n}-2u_{i,j}^{n}+u_{i,j-1}^{n}}{h_{2}^{2}}-\beta_{11}\frac{T_{i+1,j}^{n}-T_{i-1,j}^{n}}{2h_{1}}=\rho\frac{u_{i,j}^{n+1}-2u_{i,j}^{n}+u_{i,j}^{n-1}}{\tau^{2}} \quad (20)$$

The difference equation (20) can be reduced to the following tridiagonal form

$$a_{i}u_{i+1,j}^{n+1}+b_{i}u_{i,j}^{n+1}+c_{i}u_{i-1,j}^{n+1}=f_{i} \quad (21)$$

where $a_{i}=\frac{C_{1111}}{h_{1}^{2}}$, $b_{i}=-2(\frac{C_{1111}}{h_{1}^{2}}+\frac{\rho}{\tau^{2}})$, $c_{i}=\frac{C_{1111}}{h_{1}^{2}}$

$$f_{i}=\rho\frac{-2u_{i,j}^{n}+u_{i,j}^{n-1}}{\tau^{2}}-\left(C_{1122}+C_{1212}\right)\frac{v_{i+1,j+1}^{n}-v_{i-1,j+1}^{n}-v_{i+1,j-1}^{n}+v_{i-1,j-1}^{n}}{4h_{1}h_{2}}-$$
$$-C_{1212}\frac{u_{i,j+1}^{n}-2u_{i,j}^{n}+u_{i,j-1}^{n}}{h_{2}^{2}}+\beta_{11}\frac{T_{i+1,j}^{n}-T_{i-1,j}^{n}}{2h_{1}}$$

Equation (21) together with the boundary conditions $u_{i}\big|_{\Sigma_1}=u_{i}^{0}$ is solved by the elimination method. By changing $j=\overline{1,N-1}$ we find the values of displacements u on the layer $(k+1)$. In the same way, we calculate the displacement values v, and for the temperature T, this calculation method is applied starting from the first layer.

4. Numerical Tests

The initial (5) and boundary (6) conditions, in the case of a two-dimensional orthotropic rectangle, take the following form:

$$u(x,y,t)\big|_{t=0}=0,\quad \frac{\partial u}{\partial t}\bigg|_{t=0}=0,\quad v(x,y,t)\big|_{t=0}=0,\quad \frac{\partial v}{\partial t}\bigg|_{t=0}=0$$

$$T(x,y,t)\big|_{t=0}=T_0+T_0\sin\left(\frac{\pi x}{l_1}\right)\sin\left(\frac{\pi y}{l_2}\right),$$

$$u(x,y,t)\big|_{x=0}=0,\ \ u(x,y,t)\big|_{x=\ell_1}=0,\ v(x,y,t)\big|_{x=0}=0,\quad v(x,y,t)\big|_{x=\ell_2}=0,$$

$$u(x,y,t)\big|_{y=0}=0,\ \ u(x,y,t)\big|_{y=\ell_2}=0\ \ v(x,y,t)\big|_{y=0}=0,\quad v(x,y,t)\big|_{y=\ell_2}=0,$$

$$T(x,y,t)\big|_{x=0}=T_0,\quad T(x,y,t)\big|_{x=l_1}=T_0,$$

$$T(x,y,t)\big|_{y=0}=T_0,\quad T(x,y,t)\big|_{y=l_2}=T_0$$

with the following constants

$\lambda_{11}=0.06$, $\lambda_{22}=0.03$, $C_{1111}=0.78$, $C_{2222}=0.3$, $C_{1122}=0.44$, $C_{1212}=0.5$,
$\rho=0.86$, $C_\varepsilon=3.4$, $T_0=15$, $h_1=0.1$, $h_2=0.1$, $\tau=0.01$, $\ell_1=1$, $\ell_2=1$.

Table 1. Values of the function *u (x, y , t)* (explicit scheme) at *t = 0.08*

	x=0	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5	x=0.6	x=0.7	x=0.8	x=0.9	x=1
y=0	0	0	0	0	0	0	0	0	0	0	0
y=0.1	0	-0.00025	-0.00022	-0.00016	-0.00008	0	0.00008	0.00016	0.00022	0.00025	0
y=0.2	0	-0.00045	-0.00040	-0.00029	-0.00015	0	0.00015	0.00029	0.00040	0.00045	0
y=0.3	0	-0.00062	-0.00055	-0.00040	-0.00021	0	0.00021	0.00040	0.00055	0.00062	0
y=0.4	0	-0.00072	-0.00065	-0.00047	-0.00025	0	0.00025	0.00047	0.00065	0.00072	0
y=0.5	0	-0.00076	-0.00068	-0.00049	-0.00026	0	0.00026	0.00049	0.00068	0.00076	0
y=0.6	0	-0.00072	-0.00065	-0.00047	-0.00025	0	0.00025	0.00047	0.00065	0.00072	0
y=0.7	0	-0.00062	-0.00055	-0.00040	-0.00021	0	0.00021	0.00040	0.00055	0.00062	0
y=0.8	0	-0.00045	-0.00040	-0.00029	-0.00015	0	0.00015	0.00029	0.00040	0.00045	0
y=0.9	0	-0.00025	-0.00022	-0.00016	-0.00008	0	0.00008	0.00016	0.00022	0.00025	0
y=1	0	0	0	0	0	0	0	0	0	0	0

Table 2. Values of the function *u (x, y , t)* (elimination method) at *t = 0.08*

	x=0	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5	x=0.6	x=0.7	x=0.8	x=0.9	x=1
y=0	0	0	0	0	0	0	0	0	0	0	0
y=0.1	0	-0.00024	-0.00022	-0.00016	-0.00008	0	0.00008	0.00016	0.00022	0.00024	0
y=0.2	0	-0.00044	-0.00040	-0.00029	-0.00015	0	0.00015	0.00029	0.00040	0.00044	0
y=0.3	0	-0.00060	-0.00055	-0.00040	-0.00021	0	0.00021	0.00040	0.00055	0.00060	0
y=0.4	0	-0.00071	-0.00065	-0.00047	-0.00025	0	0.00025	0.00047	0.00065	0.00071	0
y=0.5	0	-0.00074	-0.00068	-0.00050	-0.00026	0	0.00026	0.00050	0.00068	0.00074	0
y=0.6	0	-0.00071	-0.00065	-0.00047	-0.00025	0	0.00025	0.00047	0.00065	0.00071	0
y=0.7	0	-0.00060	-0.00055	-0.00040	-0.00021	0	0.00021	0.00040	0.00055	0.00060	0
y=0.8	0	-0.00044	-0.00040	-0.00029	-0.00015	0	0.00015	0.00029	0.00040	0.00044	0
y=0.9	0	-0.00024	-0.00022	-0.00016	-0.00008	0	0.00008	0.00016	0.00022	0.00024	0
y=1	0	0	0	0	0	0	0	0	0	0	0

Table 3. Values of the function *v (x, y , t)* (explicit scheme) at *t = 0.08*

	x=0	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5	x=0.6	x=0.7	x=0.8	x=0.9	x=1
y=0	0	0	0	0	0	0	0	0	0	0	0
y=0.1	0	-0.00039	-0.00072	-0.00099	-0.00117	-0.00123	-0.00117	-0.00099	-0.00072	-0.00039	0
y=0.2	0	-0.00034	-0.00064	-0.00089	-0.00104	-0.00109	-0.00104	-0.00089	-0.00064	-0.00034	0
y=0.3	0	-0.00025	-0.00047	-0.00064	-0.00076	-0.00080	-0.00076	-0.00064	-0.00047	-0.00025	0
y=0.4	0	-0.00013	-0.00025	-0.00034	-0.00040	-0.00042	-0.00040	-0.00034	-0.00025	-0.00013	0
y=0.5	0	0	0	0	0	0	0	0	0	0	0
y=0.6	0	0.00013	0.00025	0.00034	0.00040	0.00042	0.00040	0.00034	0.00025	0.00013	0
y=0.7	0	0.00025	0.00047	0.00064	0.00076	0.00080	0.00076	0.00064	0.00047	0.00025	0
y=0.8	0	0.00034	0.00064	0.00089	0.00104	0.00109	0.00104	0.00089	0.00064	0.00034	0
y=0.9	0	0.00039	0.00072	0.00099	0.00117	0.00123	0.00117	0.00099	0.00072	0.00039	0
y=1	0	0	0	0	0	0	0	0	0	0	0

Table 4. Values of the function *v (x, y , t)* (elimination method) at *t = 0.08*

	x=0	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5	x=0.6	x=0.7	x=0.8	x=0.9	x=1
y=0	0	0	0	0	0	0	0	0	0	0	0
y=0.1	0	-0.00039	-0.00072	-0.00099	-0.00117	-0.00123	-0.00117	-0.00099	-0.00072	-0.00039	0
y=0.2	0	-0.00034	-0.00065	-0.00089	-0.00104	-0.00110	-0.00104	-0.00089	-0.00065	-0.00034	0
y=0.3	0	-0.00025	-0.00047	-0.00065	-0.00076	-0.00080	-0.00076	-0.00065	-0.00047	-0.00025	0
y=0.4	0	-0.00013	-0.00025	-0.00034	-0.00040	-0.00042	-0.00040	-0.00034	-0.00025	-0.00013	0
y=0.5	0	0	0	0	0	0	0	0	0	0	0
y=0.6	0	0.00013	0.00025	0.00034	0.00040	0.00042	0.00040	0.00034	0.00025	0.00013	0
y=0.7	0	0.00025	0.00047	0.00065	0.00076	0.00080	0.00076	0.00065	0.00047	0.00025	0
y=0.8	0	0.00034	0.00065	0.00089	0.00104	0.00110	0.00104	0.00089	0.00065	0.00034	0
y=0.9	0	0.00039	0.00072	0.00099	0.00117	0.00123	0.00117	0.00099	0.00072	0.00039	0
y=1	0	0	0	0	0	0	0	0	0	0	0

Table 5. Values of the function *T (x, y , t)* (explicit scheme) at *t = 0.08*

	x=0	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5	x=0.6	x=0.7	x=0.8	x=0.9	x=1
y=0	15	15	15	15	15	15	15	15	15	15	15
y=0.1	15	16.40181	17.66636	18.66994	19.31427	19.53630	19.31427	18.66994	17.66636	16.40181	15
y=0.2	15	17.66627	20.07145	21.98024	23.20575	23.62804	23.20575	21.98024	20.07145	17.66627	15
y=0.3	15	18.66981	21.98023	24.60743	26.29420	26.87542	26.29420	24.60743	21.98023	18.66981	15
y=0.4	15	19.31412	23.20574	26.29420	28.27710	28.96036	28.27710	26.29420	23.20574	19.31412	15
y=0.5	15	19.53613	23.62803	26.87542	28.96036	29.67879	28.96036	26.87542	23.62803	19.53613	15
y=0.6	15	19.31412	23.20574	26.29420	28.27710	28.96036	28.27710	26.29420	23.20574	19.31412	15
y=0.7	15	18.66981	21.98023	24.60743	26.29420	26.87542	26.29420	24.60743	21.98023	18.66981	15
y=0.8	15	17.66627	20.07145	21.98024	23.20575	23.62804	23.20575	21.98024	20.07145	17.66627	15
y=0.9	15	16.40181	17.66636	18.66994	19.31427	19.53630	19.31427	18.66994	17.66636	16.40181	15
y=1	15	15	15	15	15	15	15	15	15	15	15

Table 6. Values of the function *T (x, y , t)* (elimination method) at *t = 0.08*

	x=0	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5	x=0.6	x=0.7	x=0.8	x=0.9	x=1
y=0	15	15	15	15	15	15	15	15	15	15	15
y=0.1	15	16.42014	17.70121	18.71790	19.37065	19.59558	19.37065	18.71790	17.70121	16.42014	15
y=0.2	15	17.70113	20.13777	22.07153	23.31308	23.74089	23.31308	22.07153	20.13777	17.70113	15
y=0.3	15	18.71778	22.07153	24.73312	26.44197	27.03080	26.44197	24.73312	22.07153	18.71778	15
y=0.4	15	19.37051	23.31308	26.44197	28.45084	29.14305	28.45084	26.44197	23.31308	19.37051	15
y=0.5	15	19.59543	23.74089	27.03080	29.14305	29.87088	29.14305	27.03080	23.74089	19.59543	15
y=0.6	15	19.37051	23.31308	26.44197	28.45084	29.14305	28.45084	26.44197	23.31308	19.37051	15
y=0.7	15	18.71778	22.07153	24.73312	26.44197	27.03080	26.44197	24.73312	22.07153	18.71778	15
y=0.8	15	17.70113	20.13777	22.07153	23.31308	23.74089	23.31308	22.07153	20.13777	17.70113	15
y=0.9	15	16.42014	17.70121	18.71790	19.37065	19.59558	19.37065	18.71790	17.70121	16.42014	15
y=1	15	15	15	15	15	15	15	15	15	15	15

a) Explicit scheme **b) Implicit scheme**

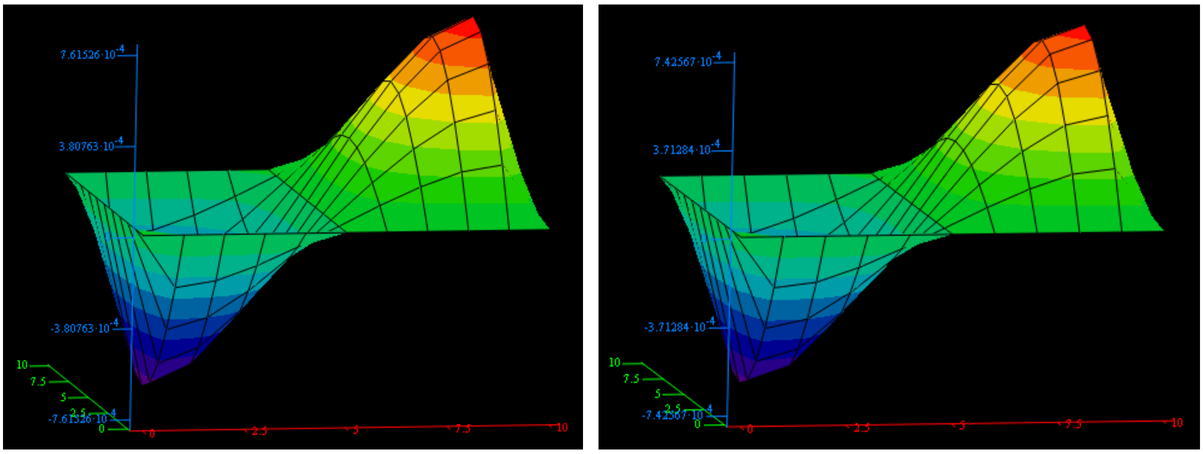

Fig. 1 a, b. Distribution of displacement *u (x, y, t)* in an orthotropic rectangle with $t = 0.08$

a) Explicit scheme **b) Implicit scheme**

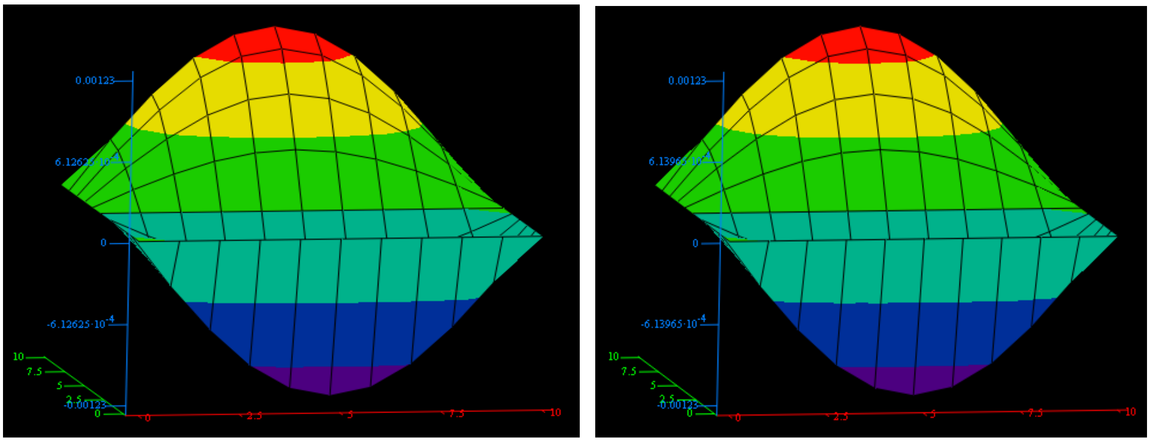

Fig. 2 a, b. Distribution of displacement *v (x, y , t)* in an orthotropic rectangle with $t = 0.08$

a) Explicit scheme **b) Implicit scheme**

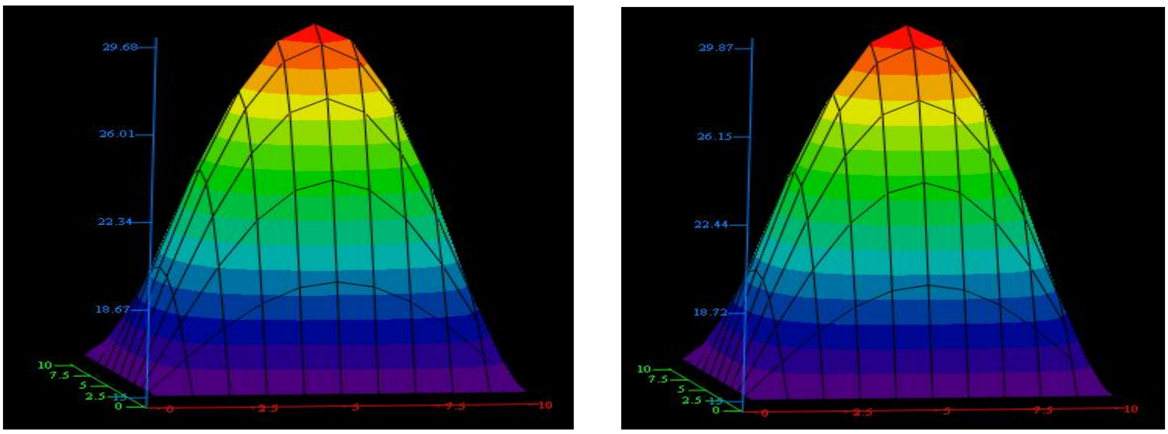

Fig. 3 a, b. Temperature distribution *T (x, y , t)* in an orthotropic rectangle at $t = 0.1$

CONCLUSION

Tables 1-6 show the numerical results of a two-dimensional coupled dynamic problem of an orthotropic rectangle. According to the initial and boundary conditions (22), a clamped rectangle is initially applied with a temperature field. The elastic constants of an orthotropic material are slightly different from an isotropic material. Deforming process mainly comes from - the thermal field. According to tables 5-6, you can see that the highest temperature is reached in the center of the rectangle and pa , but, respectively, 29.679 and 29.871 according to the explicit scheme and the elimination method. From Tables 1–4, it can be seen that the displacement values obtained by the two mentioned methods almost coincide; this ensures the validity of the numerical results obtained.

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